Exponential Distribution

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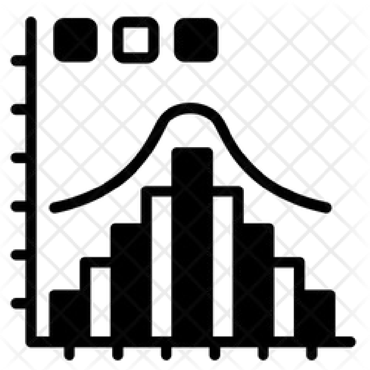


Exponential Distribution

- The exponential distribution (also called the negative exponential distribution) is a probability distribution that describes time between events in a Poisson process.
- There is a strong relationship between the Poisson distribution and the Exponential distribution.
- For example, let’s say a Poisson distribution models the number of births in a given time period. The time in between each birth can be modeled with an exponential distribution (Young & Young, 1998).

Poisson Process

- The Poisson process gives you a way to find probabilities for random points in time for a process. A “process” could be almost anything:
  - Accidents at an interchange.
  - File requests on a server.
  - Customers arriving at a store.
  - Battery failure and replacement.

Poisson Process

- The Poisson process can tell you when one of these random points in time will likely happen.
- For example, when customers will arrive at a store, or when a battery might need to be replaced.
- It’s basically a counting process; it counts the number of times an event has occurred since a given point in time, like 1210 customers since 1 p.m., or 543 files since noon.
- An assumption for the process is that it is only used for independent events.

Use of Exponential Distribution

- The exponential distribution is mostly used for testing product reliability.
- It's also an important distribution for building continuous-time Markov chains.
- The exponential often models waiting times and can help you to answer questions like:
  - "How much time will go by before a major hurricane hits the Atlantic Seaboard?" or
  - "How long will the transmission in my car last before it breaks?".

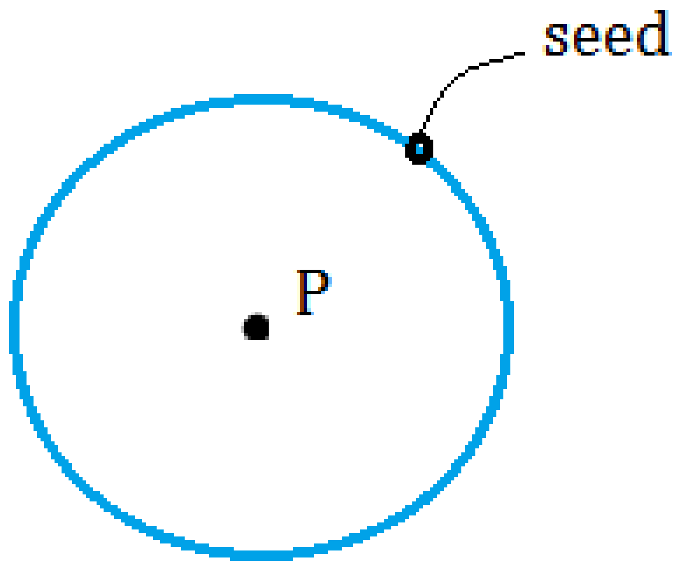
Use of Exponential Distribution

- If you assume that the answer to these questions is unknown, you can think of the elapsed time as a random variable with an exponential distribution as long as the events occur continuously and independently at a constant rate.

Use of Exponential Distribution

- Young and Young (198) give an example of how the exponential distribution can also model space between events (instead of time between events).
- Say that grass seeds are randomly dispersed over a field; Each point in the field has an equal chance of a seed landing there.
- Now select a point P, then measure the distance to the nearest seed. This distance becomes the radius for a circle with point P at the center. The circle’s area has an exponential distribution.

Use of Exponential Distribution



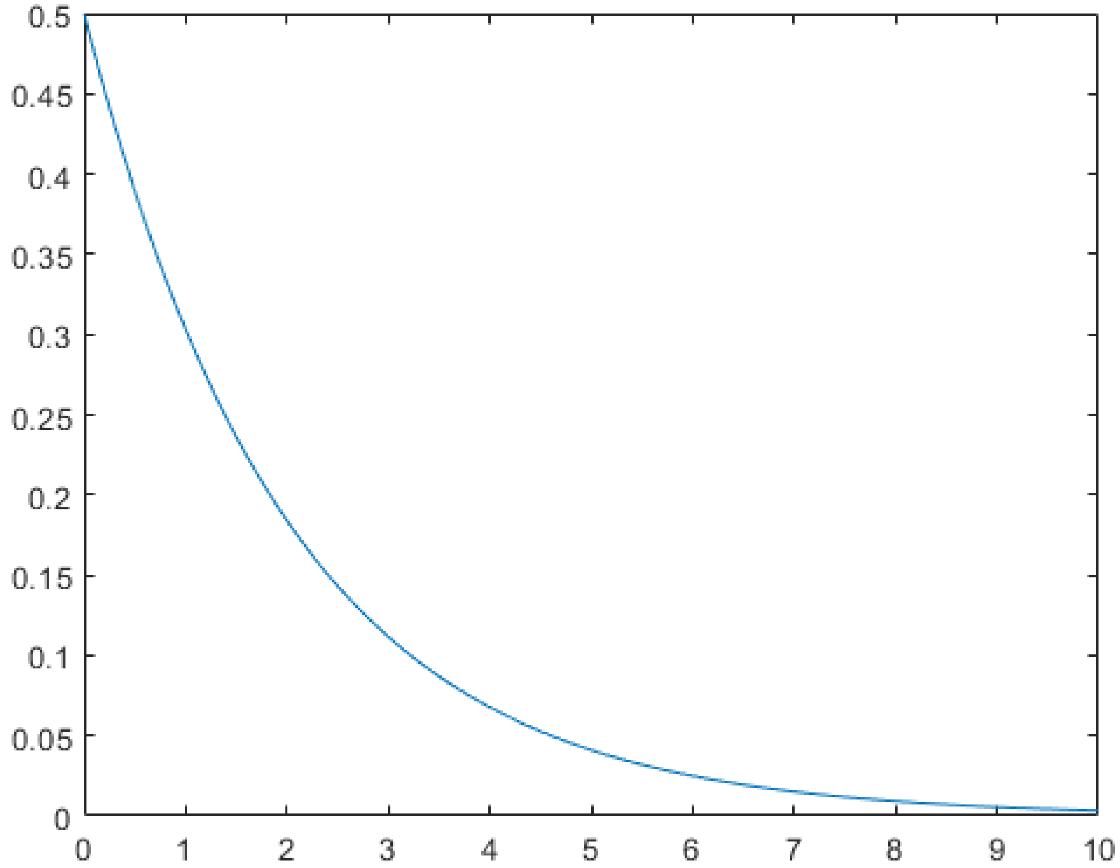
Memoryless Property

- This distribution has a memoryless property, which means it “forgets” what has come before it.
- In other words, if you continue to wait, the length of time you wait neither increases nor decreases the probability of an event happening. Any time may be marked down as time zero.
- Let’s say a hurricane hits your island. The probability of another hurricane hitting in one week, one month, or ten years from that point are all equal. The exponential is the only distribution with the memoryless property.

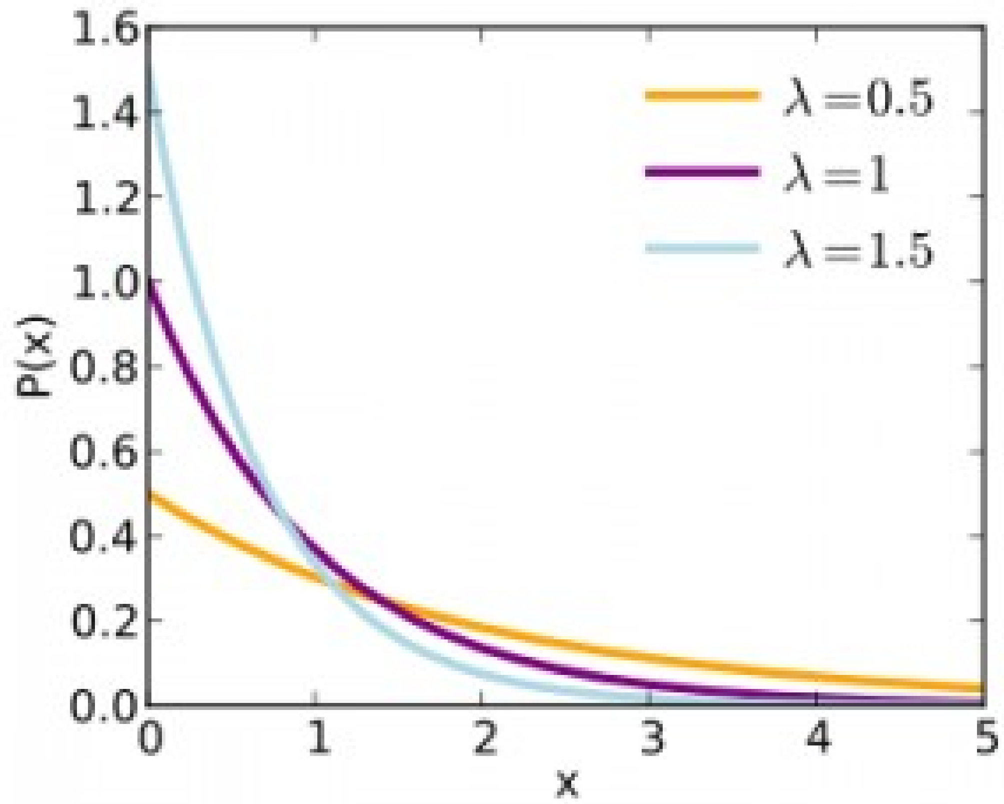
Memoryless Property

- While many authors consider the memoryless property to be one of the most useful aspects of the distribution, it's also the reason why it may not make sense for modeling certain situations.
- For example, let's say you wanted to model deaths of family dogs. Because of the memoryless property, the probability of a pet dog dying at age 1 would be the same as the dog dying at age 15, which is obviously nonsensical.
- Therefore, you should think about whether the exponential makes sense logically to your particular area of interest. For lifetime studies, the exponential is usually used only as a first rough model for the process.

Graph



Probability Density Function



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Probability Density Function

- The most common form of the pdf is:

  $F(x;\lambda) = e^{-\lambda x}$ $x > 0$.
- Where:
  - e = the natural number e,
  - λ = mean time between events,
  - x = a random variable.
- For x less than 0, $F(x;\lambda) = 0$

Probability Density Function

- For x less than 0, $F(x;\lambda) = 0$
- An alternate form of the pdf (Engineering Statistics Handbook) is:
- $f(x) = (1/\beta)\, e^{-(x-\mu)/\beta}$ $x \geq \mu$; $\beta > 0$

  where μ is the location parameter and β is the scale parameter. The scale parameter is sometimes referred to as λ which equals $1/\beta$.
- If $\mu = 0$ and $\beta = 1$ it is called the standard exponential distribution and has the equation:

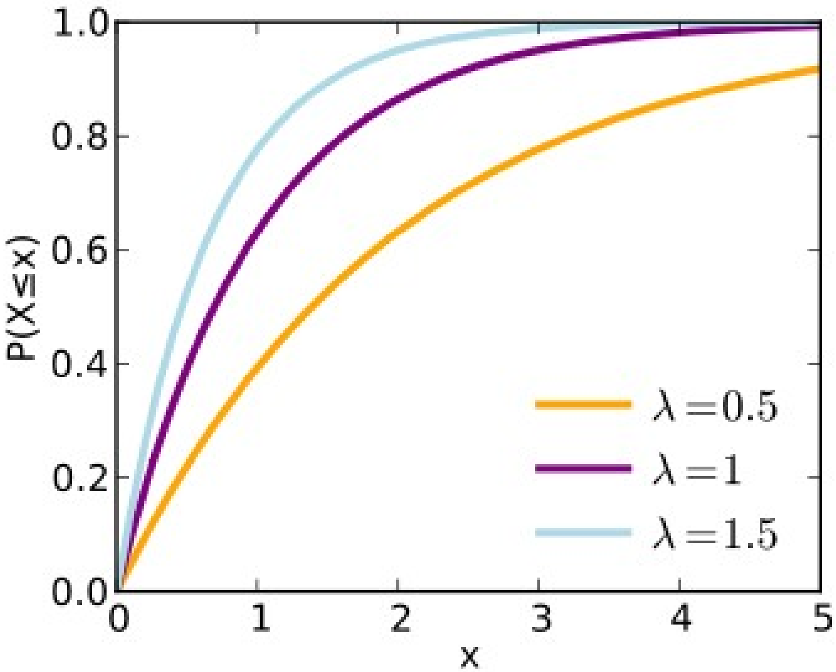
  $f(x)=e^{-x}$ for $x \geq 0$.

Cumulative Distribution Function

- The formula for the cumulative distribution function of the exponential distribution is:

  $F(x) = 1 - e^{-\lambda x}$

  $x \geq 0; \lambda > 0$



Connection to Other Distributions

- The exponential distribution is related to the Poisson distribution.
- While the exponential models time between successive events over a continuous time interval, the Poisson distribution deals with events that happen over a fixed period of time.
- The number of times the event will happen within a certain amount of time is a Poisson distribution if:
  - The event can happen more than one time.
  - If the time elapsed between two events is exponentially distributed.
  - The events are independent of previous events.

Using Python

- You can generate an exponentially distributed random variable using scipy.stats module's expon.rvs() method which takes shape parameter scale as its argument which is nothing but 1/lambda in the equation.
- To shift distribution use the loc argument, size decides the number of random variates in the distribution.
- If you want to maintain reproducibility, include a random_state argument assigned to a number.

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