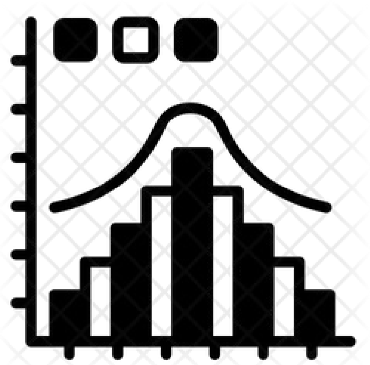
Weibull Distribution



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Weibull Distribution

- The Weibull distribution is a continuous probability distribution named after Swedish mathematician Waloddi Weibull.
- He originally proposed the distribution as a model for material breaking strength, but recognized the potential of the distribution in his 1951 paper A Statistical Distribution Function of Wide Applicability.
- Today, it's commonly used to assess product reliability, analyze life data and model failure times. The Weibull can also fit a wide range of data from many other fields, including: biology, economics, engineering sciences, and hydrology

Weibull Distribution

- Although it's extremely useful in most cases, the Weibull isn't an appropriate model for every situation.
- For example, chemical reactions and corrosion failures are usually modeled with the lognormal distribution.

PDF

- Two versions of the Weibull probability density function (pdf) are in common use: the two parameter pdf and the three parameter pdf.
- Different authors use different notation, which makes the notation a little confusing if you're looking at different texts.
- For example, The Engineering Statistics Handbook uses gamma(γ) to represent the shape parameter, while other authors (e.g. Fritz Scholz, writing for Boeing) use beta (β).

Three parameter Weibull

- The formula for the probability density function of the three parameter general Weibull distribution is:

$$f(x) = \frac{\gamma}{\alpha} \left(\frac{x - \mu}{\alpha}\right)^{(\gamma - 1)} \exp\left(-\left((x - \mu)/\alpha\right)^{\gamma}\right) \qquad x \geq \mu ;\ \gamma ,\ \alpha > 0,$$

- Where:
  - γ is the shape parameter (also known as the Weibull slope or the threshold parameter). Note: some authors use β, m, or k.
  - α is the scale parameter, also called the characteristic life parameter. Note: some authors use c, v or η instead. I found a single text (Glantz & Kissell, 2013) using γ.
  - μ is the location parameter, also called the waiting time parameter or sometimes the shift parameter. Note: μ the time to failure, is not included in the two parameter version.

Three parameter Weibull

- When $\mu = 0$ and $\alpha = 1$, the formula for the pdf reduces to:

$$f(x) = \gamma x^{(\gamma - 1)} \exp^{(-(x^{\gamma})} \qquad x \geq 0; \gamma > 0,$$

which is the standard Weibull distribution.

Two parameter Weibull

- The formula is practically identical to the three parameter Weibull, except that μ isn't included:

$$f(t) = \frac{\gamma}{\alpha}\left(\frac{x}{\alpha}\right)^{\gamma-1} \exp\left(-\left[\frac{x}{\alpha}\right]^{\gamma}\right) \quad x \geq 0$$

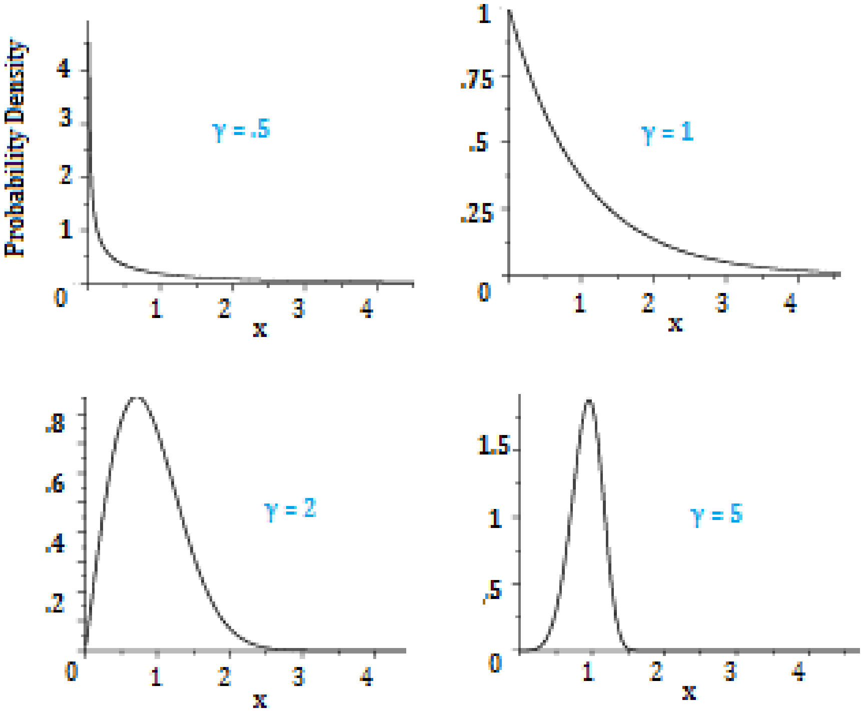
- The two parameter Weibull is often used in failure analysis, because no failure can happen before time zero.
- If you know μ, the time when the failure happens, you can subtract it from x (i.e. time t). Therefore, when you move from the two-parameter to the three-parameter version, all you have to do is replace each instance of x with (x – μ).

Gamma and Failure Rates

- The value for the shape parameter (γ) determines the failure rates:
  - If gamma is less than 1, then the failure rate decreases with time (i.e. the process has a large number of infantile or early-life failures and fewer failures as time passes).
  - For $\gamma = 1$: the failure rate is constant, which means it's indicative of useful life or random failures.
  - If $\gamma > 1$: the failure rate increases with time (i.e. the distribution models wear-out failures, which tend to happen after some time has passed).

The Weibull Family

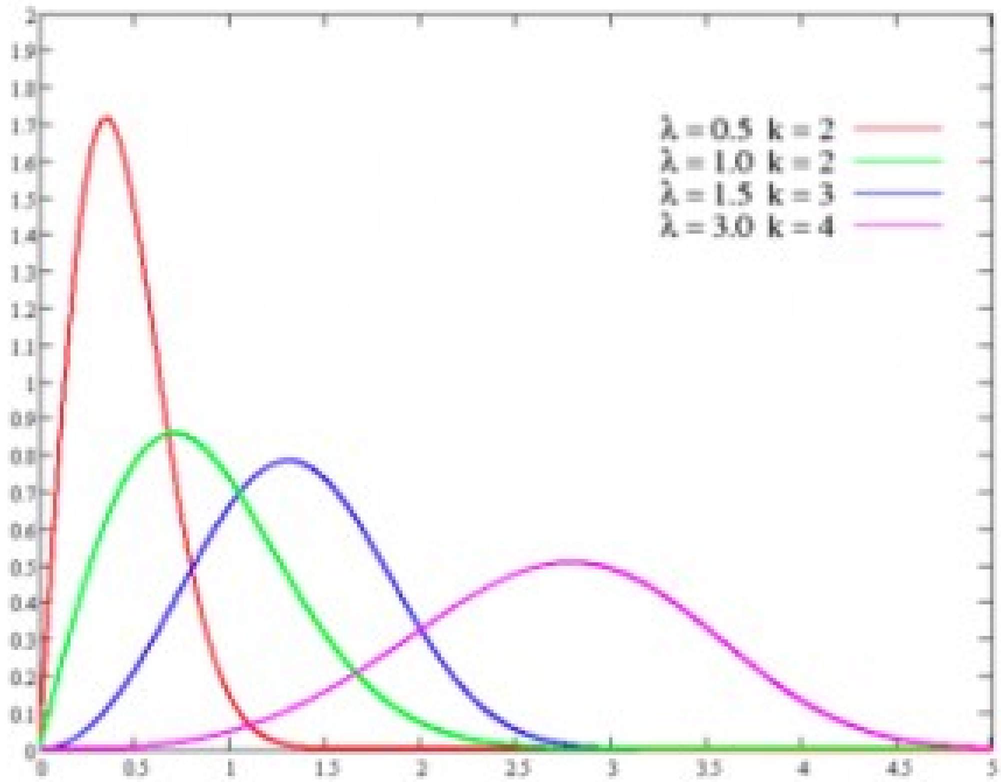
- The Weibull distribution is a family of distributions that can take on many shapes, depending on what parameters you choose.



The Weibull Family

- The Weibull distributions above include two exponential distributions (top row), a right-skewed distribution (bottom left) and a symmetric distribution (bottom right).
- The exponential distribution is a special case of the Weibull distribution, which happens when the Weibull shape parameter equals 1.

The Weibull Family



Weibull Family

- Changing α, the scale parameter, does not change the type of shape, but it does stretch out the existing shape.
- If the other two parameters are kept the same:
  - Increasing α results in the graph being stretched to the right. The height will decrease.
  - Decreasing α results in the graph being shrunk to the left (towards zero). The height will increase.

Weibull Analysis

- Weibull analysis involves using the Weibull distribution (and sometimes, the lognormal) to study life data analysis — the analysis of time to failure. For example, Weibull analysis can be used to study:
  - Lifetimes of medical and dental implants,
  - Components produced in a factory (like bearings, capacitors, or dialetrics),
  - Warranty analysis,
  - Utility services,
  - Other areas where time-to-failure is important.
- The analysis isn't limited to production; it is applicable to the design stage and in-service time as well.

Using Python

- scipy.stats.weibull_min() is a Weibull minimum continuous random variable. It is inherited from the of generic methods as an instance of the rv_continuous class. It completes the methods with details specific for this particular distribution.
- Parameters :
  - q : lower and upper tail probability
  - x : quantiles
  - loc : [optional]location parameter. Default = 0
  - scale : [optional]scale parameter. Default = 1
  - size : [tuple of ints, optional] shape or random variates.
  - moments : [optional] composed of letters [‘mvsk’]; ‘m’ = mean, ‘v’ = variance, ‘s’ = Fisher’s skew and ‘k’ = Fisher’s kurtosis. (default = ‘mv’).
  - Results : Weibull minimum continuous random variable

Thank you

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