Probability

Tushar B. Kute,
http://tusharkute.com

What is Probability?

- **Probability** is a measure of the likelihood of a random phenomenon or chance behavior.
- Probability describes the long-term proportion with which a certain **outcome** will occur in situations with short-term uncertainty.
- Example:
  - Simulate flipping a coin 100 times. Plot the proportion of heads against the number of flips. Repeat the simulation.

Probability

- Probability deals with experiments that yield random short-term results or outcomes, yet reveal long-term predictability.
- The long-term proportion with which a certain outcome is observed is the probability of that outcome.

Law of large numbers

- As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome.

Probability and event

- In probability, an **experiment** is any process that can be repeated in which the results are uncertain.
- A **simple event** is any single outcome from a probability experiment. Each simple event is denoted e_i.
- The **sample space, *S*,** of a probability experiment is the collection of all possible simple events. In other words, the sample space is a list of all possible outcomes of a probability experiment.

The event

- An **event** is any collection of outcomes from a probability experiment.
- An event may consist of one or more simple events.
- Events are denoted using capital letters such as *E*.

Example:

- Consider the probability experiment of having two children.
- (a) Identify the simple events of the probability experiment.
- (b) Determine the sample space.
- (c) Define the event E = “have one boy”.

Denoting probability

- The **probability of an event**, denoted $P(E)$, is the likelihood of that event occurring.

Properties of probabilities

- The probability of any event E, $P(E)$, must be between 0 and 1 inclusive. That is,

  $0 \leq P(E) \leq 1.$

- If an event is **impossible**, the probability of the event is 0.
- If an event is a **certainty,** the probability of the event is 1.
- If $S = \{e_1, e_2, \ldots, e_n\}$, then

  $P(e_1) + P(e_2) + \ldots + P(e_n) = 1.$

Unusual Event

- An **unusual event** is an event that has a low probability of occurring.

Method of probability

- Three methods for determining the probability of an event:

  (1) the classical method

  (2) the empirical method

  (3) the subjective method

Classical Method

- The classical method of computing probabilities requires *equally likely outcomes.*
- An experiment is said to have **equally likely outcomes** when each simple event has the same probability of occurring.

Classical Method

- If an experiment has *n* equally likely simple events and if the number of ways that an event *E* can occur is *m*, then the probability of *E*, *P*(*E*), is

$$P(E) = \frac{\text{Number of way that E can occur}}{\text{Number of Possible Outcomes}} = \frac{m}{n}$$

- So, if *S* is the sample space of this experiment, then

$$P(E) = \frac{N(E)}{N(S)}$$

Example:

- Suppose a “fun size” bag of M&Ms contains 9 brown candies, 6 yellow candies, 7 red candies, 4 orange candies, 2 blue candies, and 2 green candies. Suppose that a candy is randomly selected.

  (a) What is the probability that it is brown?

  (b) What is the probability that it is blue?

  (c) Comment on the likelihood of the candy being brown versus blue.

Emperical Method

- The probability of an event *E* is approximately the number of times event *E* is observed divided by the number of repetitions of the experiment.

$$P(E) \approx \text{relative frequency of E}$$

$$= \frac{\text{frequency of E}}{\text{number of trials of experiment}}$$

Example:

- The following data represent the number of homes with various types of home heating fuels based on a survey of 1,000 homes.
- (a) Approximate the probability that a randomly selected home uses electricity as its home heating fuel.
- (b) Would it be unusual to select a home that uses coal or coke as its home heating fuel?

Example:

HOUSE HEATING FUEL	Frequency
Utility gas	504
Bottled, tank, or LP gas	64
Electricity	307
Fuel oil, kerosene, etc.	94
Coal or coke	2
Wood	17
Solar energy	1
Other fuel	4
No fuel used	7

Subjective Probabilities

- **Subjective probabilities** are probabilities obtained based upon an educated guess.
- For example, there is a 40% chance of rain tomorrow.

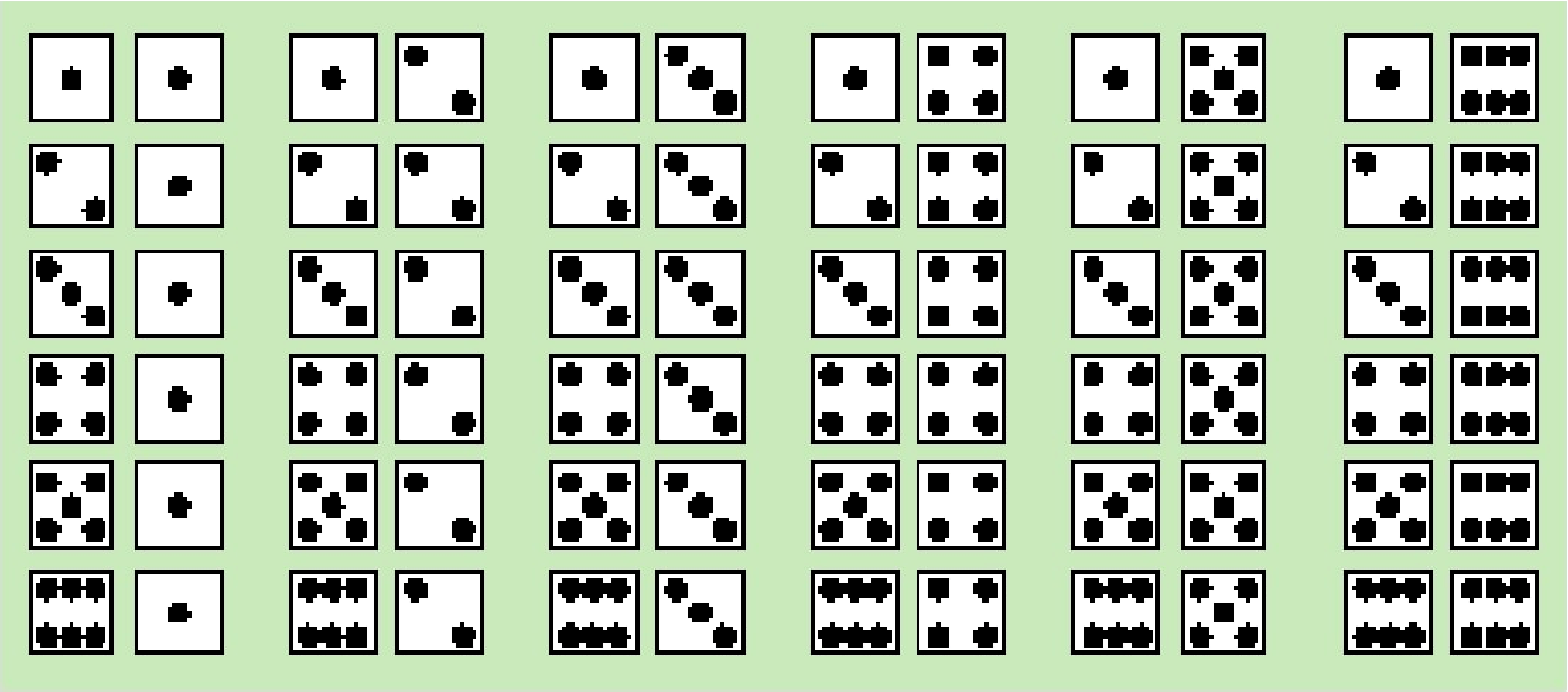
Addition Rule

- Let *E* and *F* be two events.
- ***E* and *F*** is the event consisting of simple events that belong to both *E* and *F*.
- ***E* or *F*** is the event consisting of simple events that belong to either *E* or *F* or both.

Example:

- Suppose that a pair of fair dice are thrown.
- a) Let E="rolling a seven", compute the probability of rolling a seven, i.e., P(E).
- b) Let E="rolling a two " (called 'snake eyes'), compute the probability of rolling "snake eyes", i.e., P(E).
- c) Let *E* = "the first die is a two" and let *F* = "the sum of the dice is less than or equal to 5". Find *P*(*E* or *F*) directly by counting the number of ways *E* or *F* could occur and dividing this result by the number of possible outcomes.

Example:



Conditional Probability

- The notation $P(F \mid E)$ is read "the probability of event F given event E".
- It is the probability of an event F given the occurrence of the event E.

Conditional Probability

The probability that two events *E* and *F* both occur is

$$P(E \text{ and } F) = P(E) \cdot P(F \mid E)$$

In words, the probability of *E* and *F* is the probability of event *E* occurring times the probability of event *F* occurring given the occurrence of event *E*.

Example

- The probability that a randomly selected murder victim was male is 0.7515.
- The probability that a randomly selected murder victim was less than 18 years old given that he was male was 0.1020.
- What is the probability that a randomly selected murder victim is male and is less than 18 years old?

Data based on information obtained from the United States Federal Bureau of Investigation.

- $P(\text{male and} <18)=p(<18)*P(\text{male}|<18)$
- $P(\text{male and} <18)=p(\text{male})*P(<18|\text{male})$ $=0.7515*0.1020=0.076653$

Example

- Two events *E* and *F* are **independent** if the occurrence of event *E* in a probability experiment does not affect the probability of event *F*.
- Two events are **dependent** if the occurrence of event *E* in a probability experiment affects the probability of event *F*.

Definition of independent events

- Two events *E* and *F* are independent if and only if

$$P(F \mid E) = P(F) \text{ or } P(E \mid F) = P(E)$$

Illustrating independent events

- The probability a randomly selected murder victim is male is 0.7515. The probability a randomly selected murder victim is male given that they are less than 18 years old is 0.6751.

  Since P(male) = 0.7515 and

  P(male | < 18 years old) = 0.6751,

- the events “male” and “less than 18 years old” are not independent. In fact, knowing the victim is less than 18 years old decreases the probability that the victim is male.

Multiplication Rule

If E and F are independent events, the probability E and F both occur is

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

In words, the probability of E and F is the probability of event E times the probability of event F.

Example:

- The probability that a randomly selected female aged 60 years old will survive the year is 99.186% according to the National Vital Statistics Report, Vol. 47, No. 28. What is the probability that two randomly selected 60 year old females will survive the year?
- 99.186% * 99.186% =98.38%

Multiplication Rule

Multiplication Rule for *n* Independent Events

If events *E*, *F*, *G*, … are independent, then

$$P(E \text{ and } F \text{ and } G \text{ and } \cdots) = P(E) \cdot P(F) \cdot P(G) \cdots$$

Example:

- The probability that a randomly selected female aged 60 years old will survive the year is 99.186% according to the National Vital Statistics Report, Vol. 47, No. 28. What is the probability that four randomly selected 60 year old females will survive the year?

0.99186* 0.99186* 0.99186* 0.99186=96.78%

Example:

- Suppose we have a box full of 500 golf balls. In the box, there are 50 Titleist golf balls.
- (a) Suppose two golf balls are selected randomly without replacement. What is the probability they are both Titleists?
- (b) Suppose a golf ball is selected at random and then replaced. A second golf ball is then selected. What is the probability they are both Titleists? NOTE: When sampling with replacement, the events are independent.

Example:

- If small random samples are taken from large populations without replacement, it is reasonable to assume independence of the events.
- Typically, if the sample size is less than 5% of the population size, then we treat the events as independent.

Computing 'At Least' Probabilities

- The probability that a randomly selected female aged 60 years old will survive the year is 99.186% according to the National Vital Statistics Report, Vol. 47, No. 28.
- What is the probability that at least one of 500 randomly selected 60 year old females will die during the course of the year?

  1-P(All Survived)=1-0.99186^500=50.4%

Thank you

@mitu_skillologies

/mITuSkillologies

@mitu_group

/company/mitu-skillologies

MITUSkillologies

Web Resources
https://mitu.co.in
http://tusharkute.com

contact@mitu.co.in

tushar@tusharkute.com