Language of Propositional Logic

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Langauge

- All languages have a set of symbols, rules for constructing compound constructions out of atomic constructions, and meanings assigned to the significant units.
  - For example, the letter ‘A’ is part of English, but not part of Marathi.
  - For example, English is a Subject-Verb-Object language, while Arabic is Subject-Object-Verb.
  - For example, ‘snow’ means snow in English, but ‘schnee’ means snow in German.
- The syntax of a language is the grammar of the language.
- The semantics of a language is the meaning of the significant parts.

Langauge

- Propositional Logic, PL, is a formal language, which has a set of symbols, a syntax, and a semantics. It is not a natural language, like English.
- It is possible to translate sentences of most natural languages, such as Greek, English, German, French, etc… into PL.
- PL is a language that focuses on a small set of expressions. These expressions are the words used to connect propositions (sentences) to one another: ‘and’, ‘or’, ‘if…,then’, ‘not’, ‘if and only if’, and combinations of them.

Symbols in PL

- **Symbols**:
  - Propositional Letters:
    - P, Q, R, S, T, U, V, W, X, Y, and Z
  - Logical Operators:
    - ‘□’ arrow
    - ‘□’ broken arrow
    - ‘□’ triple bar
    - ‘□’ carrot
    - ‘□’ wedge
  - Grouping Symbols:
    - ‘(,)’ parentheses, and ‘[,]’ brackets

Object Language vs. Meta Language

- The language of PL described previously is the object-language. The object language is the actual language that is used for communicating in the language. For example, just as the word ‘simple’ is part of the object language of English, the formula ‘(P □ Q)’ is part of the object language of PL.
- The object language of PL must be distinguished from the meta-language for PL. The meta-language for PL is the language used for talking about PL. It is not part of PL, and is primarily used to describe the grammar and meaning of formulas at a level of generality.
- The meta-language variables are the lower case English letters: p, q, r,…z.

Rules for Well-formed formulas

- All propositional letters P….Z are atomic well-formed formulas.
- If p and q are well formed, then so are the following:
  - □p
  - (p □ q)
  - (p □ q)
  - (p □ q)
  - (p □ q)
- Nothing is a well-formed formula, unless it follows from (1) and (2).

The semantics of PL

- PL is a language that only focuses on propositional connectives and operators. In English the main propositional connectives are ‘and’, ‘or’, ‘not’, ‘if…, then..’, and ‘if and only if’.
- Since PL is only focused on these terms it only has a semantics for these terms.
- The semantics for PL is binary and exclusive. There are only two truth-values: T and F, and no statement is both T and F.
- It is important to note that the semantics of PL is for the logical operators of its language: ‘□’, ‘□’, ‘□’, ‘□’, and ‘□’.

Logical vs. Non-logical meaning

- Logicians try to give definitions of the propositional operators of PL that match perfectly with the logical meaning of ‘and’, ‘or’, ‘not’, ‘if…then…’, and ‘if and only if’.
- However, there are many cases in which the English use of, for example ‘and’ or ‘if…,then…’, do not match the definition given to carrot and arrow.
- In philosophy of logic one studies what the correct definition of the logical operators should be, and other questions about whether logic is binary and exclusive.

Truth Table

- In order to define the logical symbols of PL, one needs to use a truth-table.
- A truth-table is a table for visually displaying the distribution of truth and falsity across a compound formula given the basic inputs from the atomic letters.

p	***q***	
T	T	
T	F	
F	T	
F	T	

Thank you

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