Rules of Inference and Natural Deduction

Tushar B. Kute,
http://tusharkute.com

Inference

- Inference:
  - In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, so generating the conclusions from evidence and facts is termed as Inference.
- Inference rules:
  - Inference rules are the templates for generating valid arguments.
  - Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.

Inference

- In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:
  - Implication: It is one of the logical connectives which can be represented as $P \rightarrow Q$. It is a Boolean expression.
  - Converse: The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \rightarrow P$.
  - Contrapositive: The negation of converse is termed as contrapositive, and it can be represented as $\neg Q \rightarrow \neg P$.
  - Inverse: The negation of implication is called inverse. It can be represented as $\neg P \rightarrow \neg Q$.

Inference

- From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	P → Q	Q→ P	¬ Q → ¬ P	¬ P → ¬ Q.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

- Hence from the above truth table, we can prove that P → Q is equivalent to ¬ Q → ¬ P, and Q→ P is equivalent to ¬ P → ¬ Q.

Types of Inference rules

- The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $$\frac{P \rightarrow Q, \quad P}{\therefore Q}$$

Types of Inference rules

- Example:

  Statement-1: "If I am sleepy then I go to bed" ==> $P \rightarrow Q$

  Statement-2: "I am sleepy" ==> P

- Conclusion: "I go to bed." ==> Q.

  Hence, we can say that, if $P \rightarrow Q$ is true and P is true then Q will be true.

- Proof by Truth table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1 ←

Types of Inference rules

- Modus Tollens:
- The Modus Tollens rule state that if P→ Q is true and ¬ Q is true, then ¬ P will also true. It can be represented as:

Notation for Modus Tollens: $\frac{P \to Q,\quad \sim Q}{\sim P}$

- Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
- Statement-2: "I do not go to the bed."==> ~Q
- Statement-3: Which infers that "I am not sleepy" => ~P

P	Q	$\sim P$	$\sim Q$	$P \to Q$
0	0	1	1	1 ←
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

tusharkute.com

Types of Inference rules

- Hypothetical Syllogism:
- The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:
- Example:

  Statement-1: If you have my home key then you can unlock my home. P→Q

  Statement-2: If you can unlock my home then you can take my money. Q→R

  Conclusion: If you have my home key then you can take my money. P→R

Types of Inference rules

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
0	0	0	1	1	1 ←
0	0	1	1	1	1 ←
0	1	0	1	0	1
0	1	1	1	1	1 ←
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1 ←

Types of Inference rules

- Disjunctive Syllogism:
- The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $$\frac{P \lor Q, \quad \neg P}{Q}$$

- Example:

Statement-1: Today is Sunday or Monday. ==>P∨Q

Statement-2: Today is not Sunday. ==> ¬P

Conclusion: Today is Monday. ==> Q

Types of Inference rules

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

Types of Inference rules

- Addition:
- The Addition rule is one the common inference rule, and it states that If P is true, then P∨Q will be true.

Notation of Addition: $\frac{P}{P \lor Q}$

- Example:

  Statement: I have a vanilla ice-cream. ==> P

  Statement-2: I have Chocolate ice-cream.

- Conclusion: I have vanilla or chocolate ice-cream. ==> (P∨Q)

  Proof by Truth-Table:

P	Q	$P \lor Q$
0	0	0
1	0	1 ←
0	1	1
1	1	1 ←

tusharkute.com

Types of Inference rules

- Simplification:
- The simplification rule state that if P∧ Q is true, then Q or P will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \land Q}{Q}$ Or $\frac{P \land Q}{P}$

- Proof by Truth-Table:

P	Q	$P \land Q$
0	0	0
1	0	0
0	1	0
1	1	1 ←

miTu skillologies

tusharkute.com

Types of Inference rules

- The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. It can be represented as

Notation of Resolution $\dfrac{P \lor Q,\quad \neg P \land R}{Q \lor R}$

- Proof by Truth-Table:

P	¬ P	Q	R	$P \lor Q$	¬ P∧R	$Q \lor R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1 ←
0	1	1	1	1	1	1 ←
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1 ←

Natural Deduction

- Normal human reasoning is generally a train of thought moving linearly from the premises to the conclusion.
- This natural process is mimicked by the "Natural" Deduction Method of Propositional Logic (also called Propositional Calculus, abbreviated PC).
- This method in PC is what is used in mathematics proofs.

Natural Deduction

- A Natural Deduction proof in PC is a sequence of wffs beginning with one or more wffs as premises; fresh premises may be added at any point in the course of a proof.
- New wffs are generated by applying "rules" to any wff or a group of wffs that have already occurred in the sequence.
- This means a Natural Deduction system has two aspects: A set of rules and a method for applying the rules. Thus it is similar to the procedure of deriving theorems in mathematics.
- So we turn the situation around and say that all mathematics proofs are instances of Natural Deduction.

Replacement Rules

De Morgan's Theorem (DM)	$\sim(p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$ $\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$
Commutation (Com.)	$(p \vee q) \Leftrightarrow (q \vee p)$ $p \wedge q \Leftrightarrow q \wedge p$
Association (Assoc.)	$[(p \vee q) \vee r] \Leftrightarrow [p \vee (q \vee r)]$ $[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$
Distribution (Dist.)	$[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$ $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
Double Negation (DN)	$p \Leftrightarrow \sim\sim p$
Material Implication (M. Imp.)	$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$
Transposition (Trans.)	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
Material Equivalence (M. Equiv.)	$(p \Leftrightarrow q) \Leftrightarrow [(p \Rightarrow q) \wedge (q \Rightarrow p)]$ $(p \Leftrightarrow q) \Leftrightarrow [(p \wedge q) \vee (\sim p \wedge \sim q)]$
Law of Exportation	$[(p \wedge q) \Rightarrow r] \Leftrightarrow [(p \Rightarrow (q \Rightarrow r)]$

Thank you

This presentation is created using LibreOffice Impress 7.0.1.2, can be used freely as per GNU General Public License

@mitu_skillologies

/mITuSkillologies

@mitu_group

/company/mitu-skillologies

MITUSkillologies

Web Resources
https://mitu.co.in
http://tusharkute.com

contact@mitu.co.in

tushar@tusharkute.com