Tableau Method

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Tableau Method

- In proof theory, the semantic tableau (also called truth tree) is a decision procedure for sentential and related logics, and a proof procedure for formulae of first-order logic.
- An analytic tableau is a tree structure computed for a logical formula, having at each node a subformula of the original formula to be proved or refuted.
- Computation constructs this tree and uses it to prove or refute the whole formula. The tableau method can also determine the satisfiability of finite sets of formulas of various logics.
- It is the most popular proof procedure for modal logics (Girle 2000).

Tableau calculi and their properties

- Proof confluence is the property of a tableau calculus to obtain a proof for an arbitrary unsatisfiable set from an arbitrary tableau, assuming that this tableau has itself been obtained by applying the rules of the calculus.
- In other words, in a proof confluent tableau calculus, from an unsatisfiable set one can apply whatever set of rules and still obtain a tableau from which a closed one can be obtained by applying some other rules.

Tableau Method

- For refutation tableaux, the objective is to show that the negation of a formula cannot be satisfied. There are rules for handling each of the usual connectives, starting with the main connective.
- In many cases, applying these rules causes the subtableau to divide into two. Quantifiers are instantiated. If any branch of a tableau leads to an evident contradiction, the branch closes. If all branches close, the proof is complete and the original formula is a logical truth.
- Although the fundamental idea behind the analytic tableau method is derived from the cut-elimination theorem of structural proof theory, the origins of tableau calculi lie in the meaning (or semantics) of the logical connectives, as the connection with proof theory was made only in recent decades.

Tableau Method

- More specifically, a tableau calculus consists of a finite collection of rules with each rule specifying how to break down one logical connective into its constituent parts.
- The rules typically are expressed in terms of finite sets of formulae, although there are logics for which we must use more complicated data structures, such as multisets, lists, or even trees of formulas. Henceforth, "set" denotes any of {set, multiset, list, tree}.

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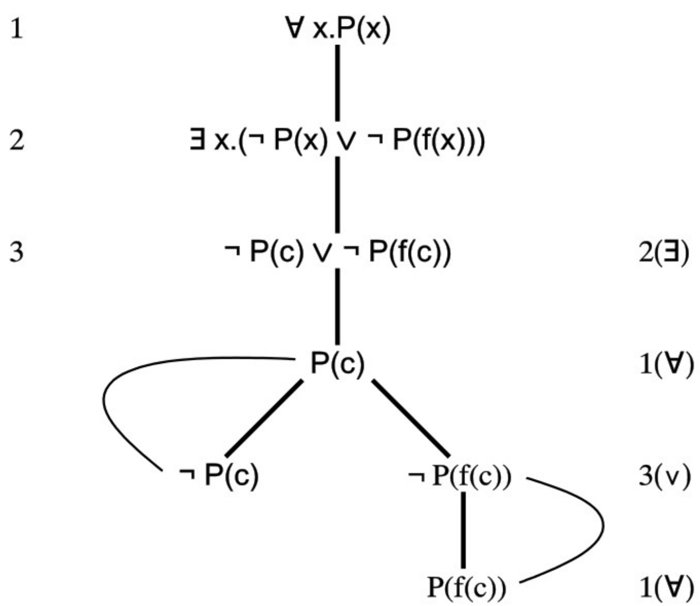
- If there is such a rule for every logical connective then the procedure will eventually produce a set which consists only of atomic formulae and their negations, which cannot be broken down any further.
- Such a set is easily recognizable as satisfiable or unsatisfiable with respect to the semantics of the logic in question. To keep track of this process, the nodes of a tableau itself are set out in the form of a tree and the branches of this tree are created and assessed in a systematic way.
- Such a systematic method for searching this tree gives rise to an algorithm for performing deduction and automated reasoning. Note that this larger tree is present regardless of whether the nodes contain sets, multisets, lists or trees.

First-order logic tableau

- Tableaux are extended to first order predicate logic by two rules for dealing with universal and existential quantifiers, respectively.
- Two different sets of rules can be used; both employ a form of Skolemization for handling existential quantifiers, but differ on the handling of universal quantifiers.
- The set of formulae to check for validity is here supposed to contain no free variables; this is not a limitation as free variables are implicitly universally quantified, so universal quantifiers over these variables can be added, resulting in a formula with no free variables.

First-order logic tableau

- A tableau without unification for {∀x.P(x), ∃x.(¬P(x)⋁¬P(f(x)))}. For clarity, formulae are numbered on the left and the formula and rule used at each step is on the right



First-order logic tableau

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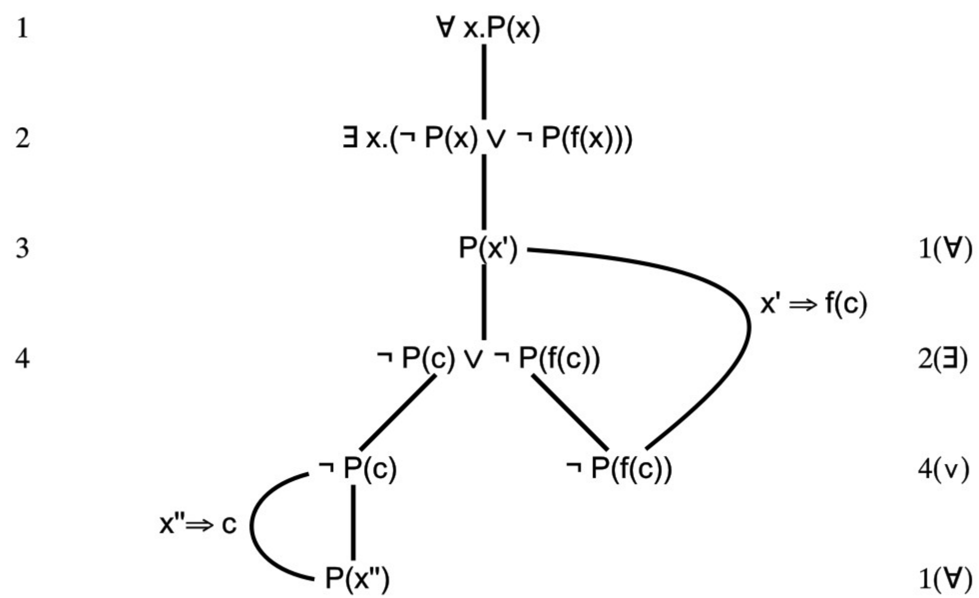

Tableau calculi and their properties

- A tableau calculus is a set of rules that allows building and modification of a tableau.
- Propositional tableau rules, tableau rules without unification, and tableau rules with unification, are all tableau calculi.
- Some important properties a tableau calculus may or may not possess are completeness, destructiveness, and proof confluence.
- A tableau calculus is called complete if it allows building a tableau proof for every given unsatisfiable set of formulae. The tableau calculi mentioned above can be proved complete.

Tableau calculi and their properties

- A remarkable difference between tableau with unification and the other two calculi is that the latter two calculi only modify a tableau by adding new nodes to it, while the former one allows substitutions to modify the existing part of the tableau.
- More generally, tableau calculi are classed as destructive or non-destructive depending on whether they only add new nodes to tableau or not.
- Tableau with unification is therefore destructive, while propositional tableau and tableau without unification are non-destructive.

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