Fuzzy Logic

Tushar B. Kute,
http://tusharkute.com

Fuzzy Logic

- Fuzzy Logic Systems (FLS) produce acceptable but definite output in response to incomplete, ambiguous, distorted, or inaccurate (fuzzy) input.
- Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning. The approach of FL imitates the way of decision making in humans that involves all intermediate possibilities between digital values YES and NO.
- The conventional logic block that a computer can understand takes precise input and produces a definite output as TRUE or FALSE, which is equivalent to human’s YES or NO.

Fuzzy Logic

- The inventor of fuzzy logic, Lotfi Zadeh, observed that unlike computers, the human decision making includes a range of possibilities between YES and NO, such as –

CERTAINLY YES
POSSIBLY YES
CANNOT SAY
POSSIBLY NO
CERTAINLY NO

- The fuzzy logic works on the levels of possibilities of input to achieve the definite output.

Implementation

- It can be implemented in systems with various sizes and capabilities ranging from small micro-controllers to large, networked, workstation-based control systems.
- It can be implemented in hardware, software, or a combination of both.

Why Fuzzy Logic?

- Fuzzy logic is useful for commercial and practical purposes.
  - It can control machines and consumer products.
  - It may not give accurate reasoning, but acceptable reasoning.
  - Fuzzy logic helps to deal with the uncertainty in engineering.

Fuzzy Logic Systems Architecture

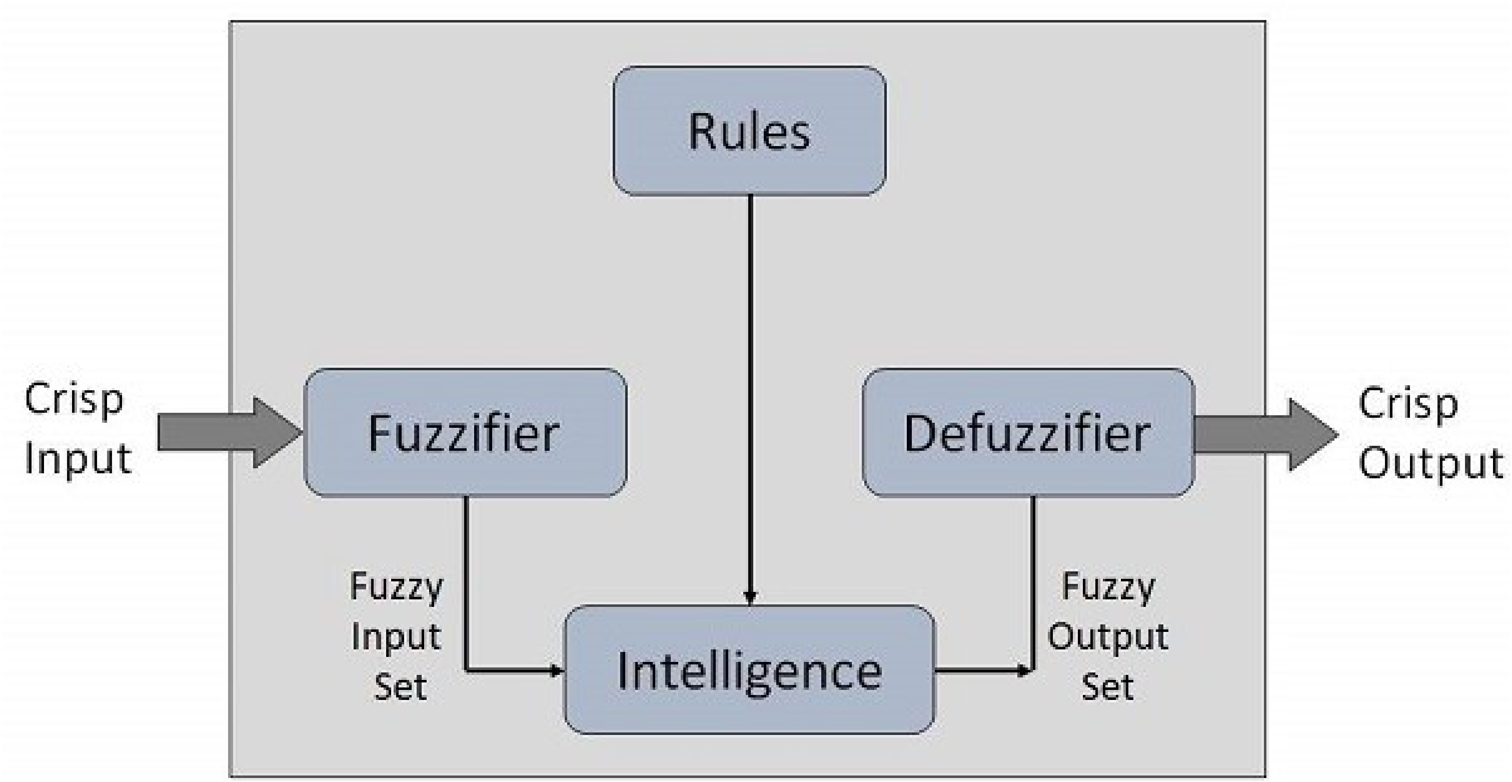
- It has four main parts as shown –
- Fuzzification Module – It transforms the system inputs, which are crisp numbers, into fuzzy sets. It splits the input signal into five steps such as –

LP	x is Large Positive
MP	x is Medium Positive
S	x is Small
MN	x is Medium Negative
LN	x is Large Negative

Fuzzy Logic Systems Architecture

- Knowledge Base – It stores IF-THEN rules provided by experts.
- Inference Engine – It simulates the human reasoning process by making fuzzy inference on the inputs and IF-THEN rules.
- Defuzzification Module – It transforms the fuzzy set obtained by the inference engine into a crisp value.

Fuzzy Logic Systems Architecture



Membership Function

- Membership functions allow you to quantify linguistic term and represent a fuzzy set graphically. A membership function for a fuzzy set A on the universe of discourse X is defined as

  $\mu_A : X \rightarrow [0,1]$.

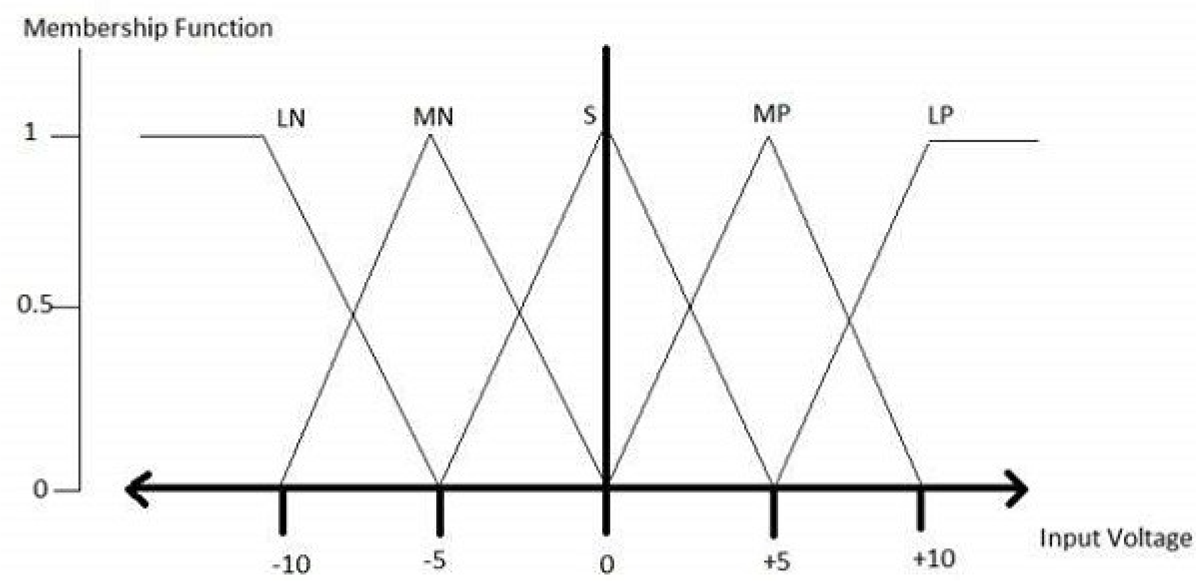
- Here, each element of X is mapped to a value between 0 and 1. It is called membership value or degree of membership. It quantifies the degree of membership of the element in X to the fuzzy set A.
  - x axis represents the universe of discourse.
  - y axis represents the degrees of membership in the [0, 1] interval.

Membership Function

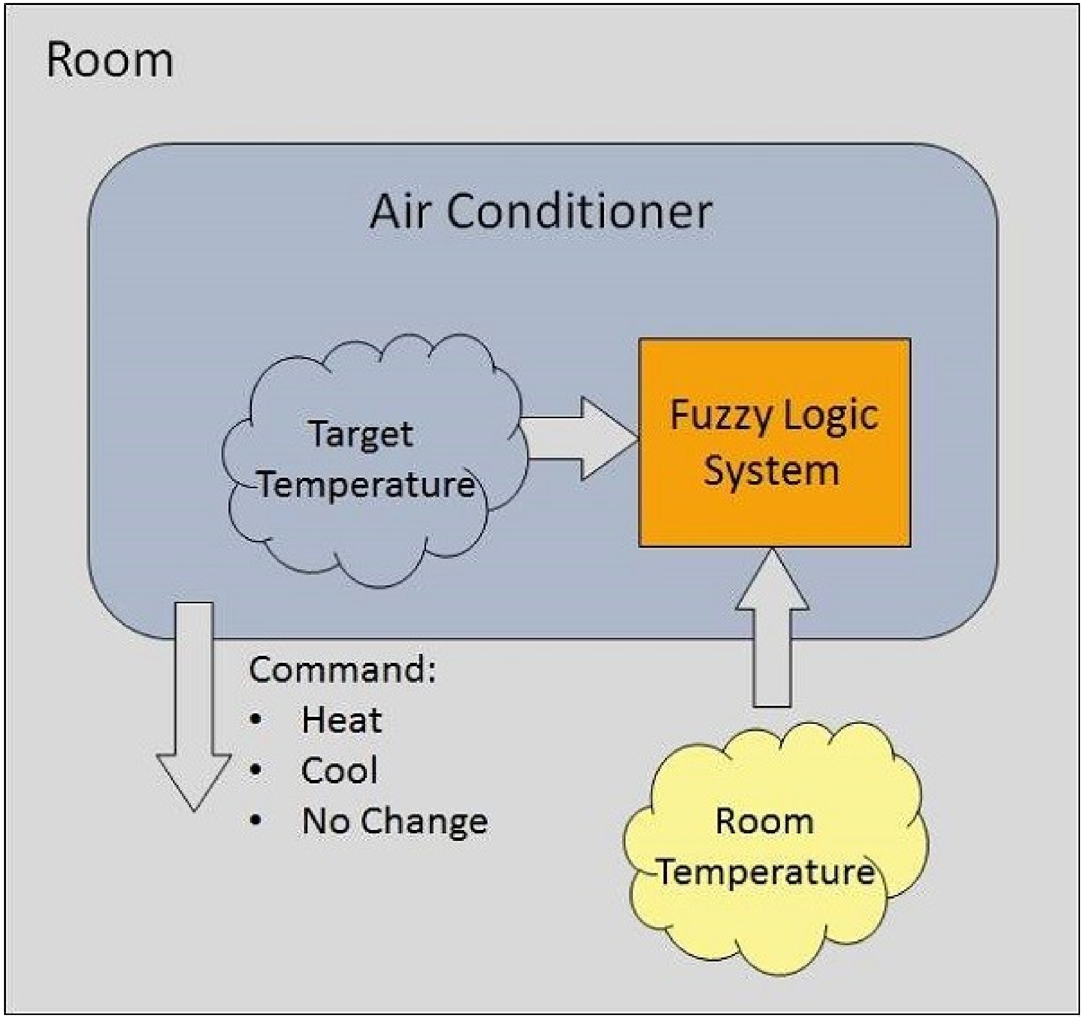
- There can be multiple membership functions applicable to fuzzify a numerical value.
- Simple membership functions are used as use of complex functions does not add more precision in the output.
- All membership functions for LP, MP, S, MN, and LN are shown as below –

Membership Function

- The triangular membership function shapes are most common among various other membership function shapes such as trapezoidal, singleton, and Gaussian.
- Here, the input to 5-level fuzzifier varies from -10 volts to +10 volts. Hence the corresponding output also changes.



Example:



Room
Air Conditioner
Target Temperature
Fuzzy Logic System
Command:
• Heat
• Cool
• No Change
Room Temperature

What is Set?

- A set is an unordered collection of different elements. It can be written explicitly by listing its elements using the set bracket.
- If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.
- Example
  - A set of all positive integers.
  - A set of all the planets in the solar system.
  - A set of all the states in India.
  - A set of all the lowercase letters of the alphabet.

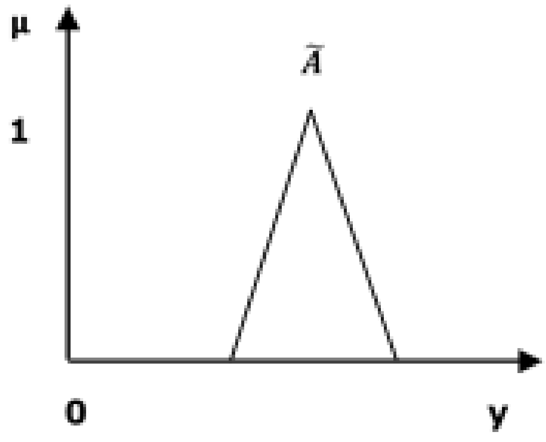
What is Set?

- Sets can be represented in two ways –
- Roster or Tabular Form
  - In this form, a set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.
  - Following are the examples of set in Roster or Tabular Form –
    - Set of vowels in English alphabet, A = {a,e,i,o,u}
    - Set of odd numbers less than 10, B = {1,3,5,7,9}

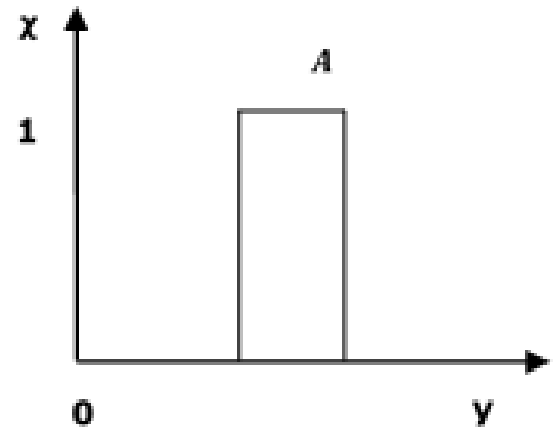
Fuzzy Set

- Fuzzy sets can be considered as an extension and gross oversimplification of classical sets. It can be best understood in the context of set membership.
- Basically it allows partial membership which means that it contain elements that have varying degrees of membership in the set.
- From this, we can understand the difference between classical set and fuzzy set.
- Classical set contains elements that satisfy precise properties of membership while fuzzy set contains elements that satisfy imprecise properties of membership.

Fuzzy Set



Membership Function of Fuzzy set Ã



Membership Function of classical set A

Fuzzy Sets

- A fuzzy set $\tilde{A}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as –

$$\tilde{A}=\{(y,\mu_{\tilde{A}}(y))|y\in U\}$$

- Here $\mu_{\tilde{A}}(y)$ = degree of membership of y in $\tilde{A}$, assumes values in the range from 0 to 1, i.e., $\mu_{\tilde{A}}(y)\in[0,1]$.

Representation Fuzzy Sets

- Let us now consider two cases of universe of information and understand how a fuzzy set can be represented.
- Case 1
  - When universe of information U is discrete and finite –

$$\widetilde{A} = \left\{ \frac{\mu_{\widetilde{A}}(y_1)}{y_1} + \frac{\mu_{\widetilde{A}}(y_2)}{y_2} + \frac{\mu_{\widetilde{A}}(y_3)}{y_3} + \dots \right\}$$

$$= \left\{ \sum_{i=1}^{n} \frac{\mu_{\widetilde{A}}(y_i)}{y_i} \right\}$$

Representation Fuzzy Sets

- Case 2
  - When universe of information U is continuous and infinite –

$$\widetilde{A} = \left\{ \int \frac{\mu_{\widetilde{A}}(y)}{y} \right\}$$

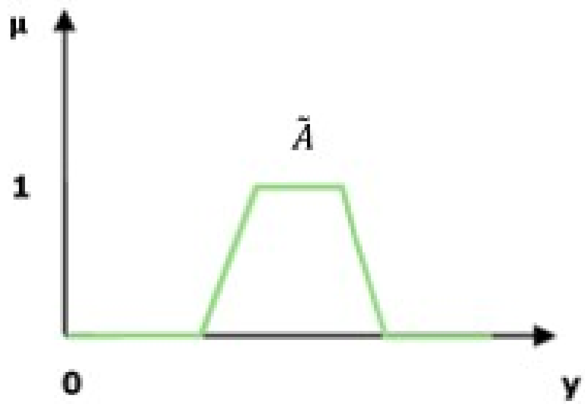
- In the above representation, the summation symbol represents the collection of each element.

Operations on Fuzzy Sets

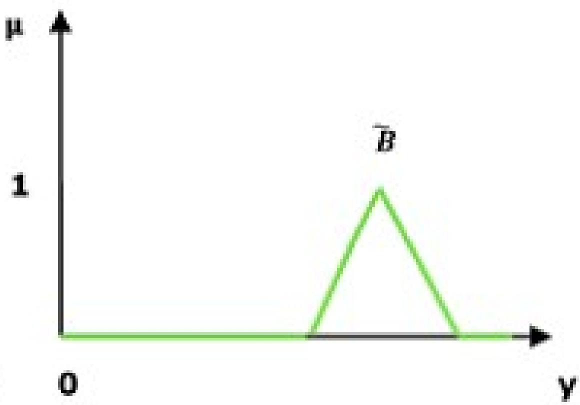
- Having two fuzzy sets A˜ and B˜, the universe of information U and an element y of the universe, the following relations express the union, intersection and complement operation on fuzzy sets.
- Union/Fuzzy 'OR'
  - Let us consider the following representation to understand how the Union/Fuzzy 'OR' relation works –

$$\mu_{\widetilde{A} \cup \widetilde{B}}(y) = \mu_{\widetilde{A}} \vee \mu_{\widetilde{B}} \quad \forall y \in U$$

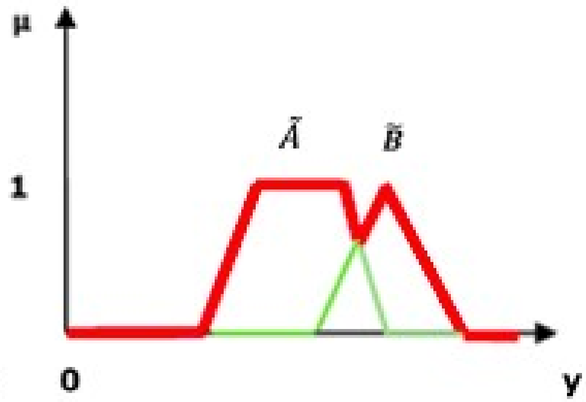
Operations on Fuzzy Sets



Fuzzy set $\tilde{A}$



Fuzzy set $\tilde{B}$



Union of two Fuzzy sets

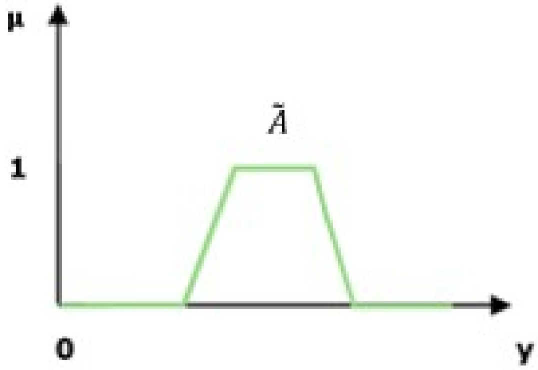
Operations on Fuzzy Sets

- Let us consider the following representation to understand how the Intersection/Fuzzy 'AND' relation works –

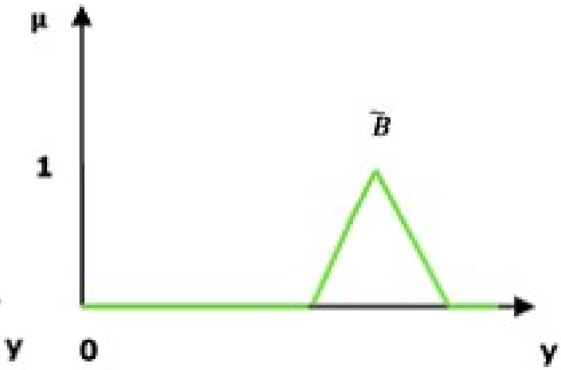
$$\mu_{\tilde{A} \cup \tilde{B}}(y) = \mu_{\tilde{A}} \vee \mu_{\tilde{B}} \quad \forall y \in U$$

- Here ∧ represents the 'min' operation.

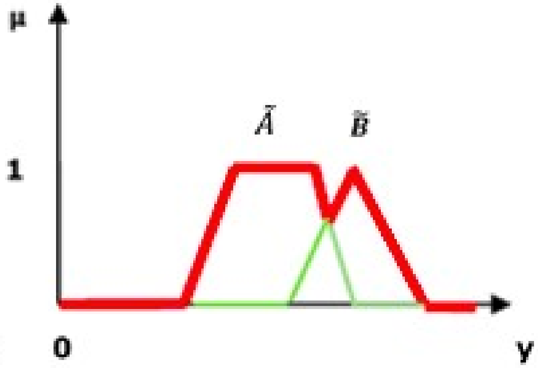
Operations on Fuzzy Sets



Fuzzy set Ã



Fuzzy set B̃



Union of two Fuzzy sets

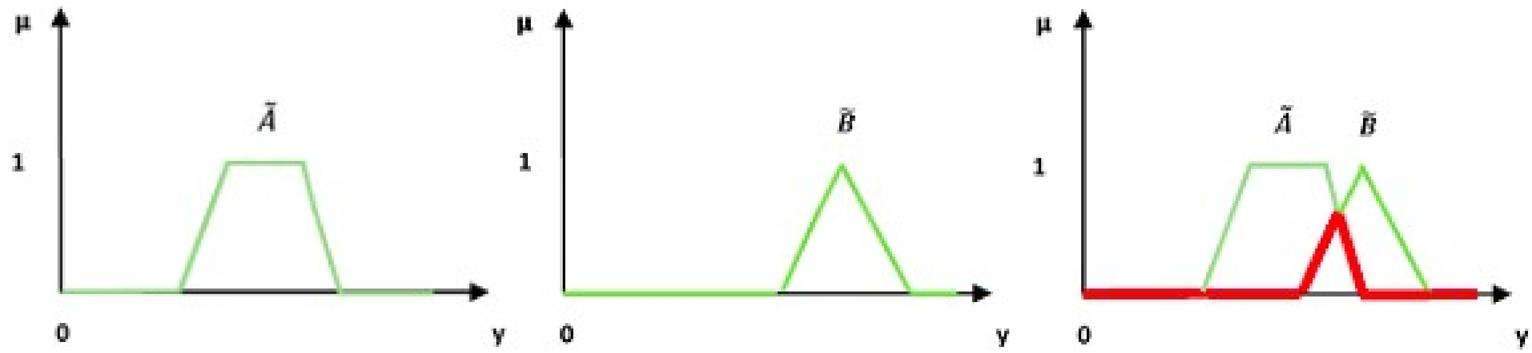
Intersection/Fuzzy 'AND'

- Let us consider the following representation to understand how the Intersection/Fuzzy 'AND' relation works –

$$\mu_{\tilde{A} \cap \tilde{B}}(y) = \mu_{\tilde{A}} \wedge \mu_{\tilde{B}} \quad \forall y \in U$$

- Here ∧ represents the 'min' operation.

Intersection/Fuzzy ‘AND’



Fuzzy set $\tilde{A}$

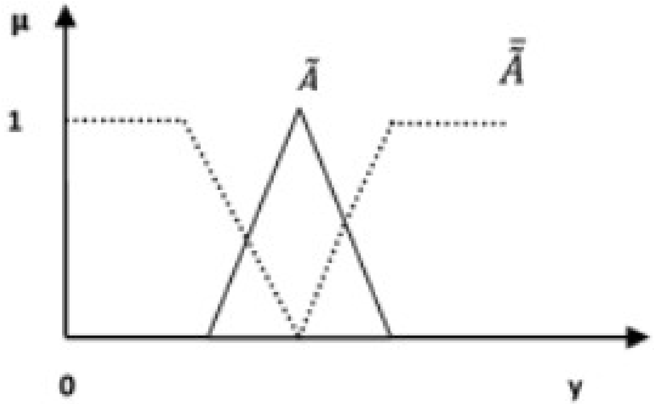
Fuzzy set $\tilde{B}$

Intersection of two Fuzzy sets

Complement/Fuzzy 'NOT'

- Let us consider the following representation to understand how the Complement/Fuzzy 'NOT' relation works –

$$\mu_{\tilde{\bar{A}}} = 1 - \mu_{\tilde{A}}(y) \quad y \in U$$



Properties of Fuzzy Sets

- Let us discuss the different properties of fuzzy sets.
- Commutative Property
  - Having two fuzzy sets A˜ and B˜, this property states –

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$$

$$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

Properties of Fuzzy Sets

- Distributive Property
  - Having three fuzzy sets A˜, B˜ and C˜, this property states –

$$\tilde{A} \cup \left(\tilde{B} \cap \tilde{C}\right) = \left(\tilde{A} \cup \tilde{B}\right) \cap \left(\tilde{A} \cup \tilde{C}\right)$$

$$\tilde{A} \cap \left(\tilde{B} \cup \tilde{C}\right) = \left(\tilde{A} \cap \tilde{B}\right) \cup \left(\tilde{A} \cap \tilde{C}\right)$$

Properties of Fuzzy Sets

- Idempotency Property
  - For any fuzzy set A˜ , this property states –

$$\widetilde{A} \cup \widetilde{A} = \widetilde{A}$$

$$\widetilde{A} \cap \widetilde{A} = \widetilde{A}$$

Properties of Fuzzy Sets

- Identity Property
  - For fuzzy set A˜ and universal set U, this property states –

$$\widetilde{A} \cup \varphi = \widetilde{A}$$

$$\widetilde{A} \cap U = \widetilde{A}$$

$$\widetilde{A} \cap \varphi = \varphi$$

$$\widetilde{A} \cup U = U$$

Properties of Fuzzy Sets

- Transitive Property
  - Having three fuzzy sets A˜ , B˜ and C˜, this property states –

$$If\ \widetilde{A} \subseteq \widetilde{B} \subseteq \widetilde{C},\ then\ \widetilde{A} \subseteq \widetilde{C}$$

Properties of Fuzzy Sets

- Involution Property
  - For any fuzzy set A˜ , this property states –

$$\overline{\overline{\tilde{A}}} = \tilde{A}$$

Properties of Fuzzy Sets

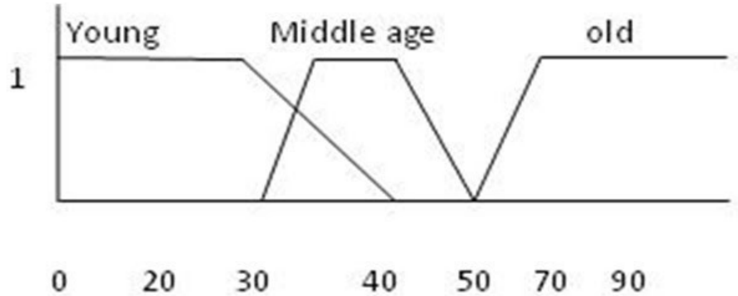
- De Morgan's Law
  - This law plays a crucial role in proving tautologies and contradiction.
  - This law states –

$$\overline{\widetilde{A} \cap \widetilde{B}} = \overline{\widetilde{A}} \cup \overline{\widetilde{B}}$$

$$\overline{\widetilde{A} \cup \widetilde{B}} = \overline{\widetilde{A}} \cap \overline{\widetilde{B}}$$

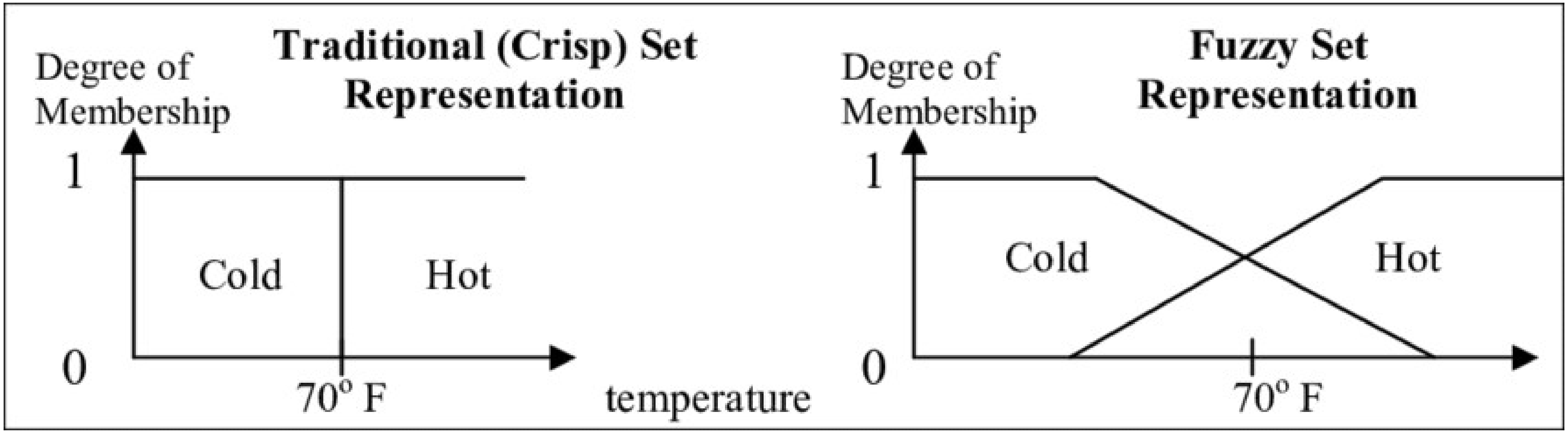
Difference:

FUZZY SET	CRISP SET
Defines value between 0 or 1	Defines either value is 0 or 1

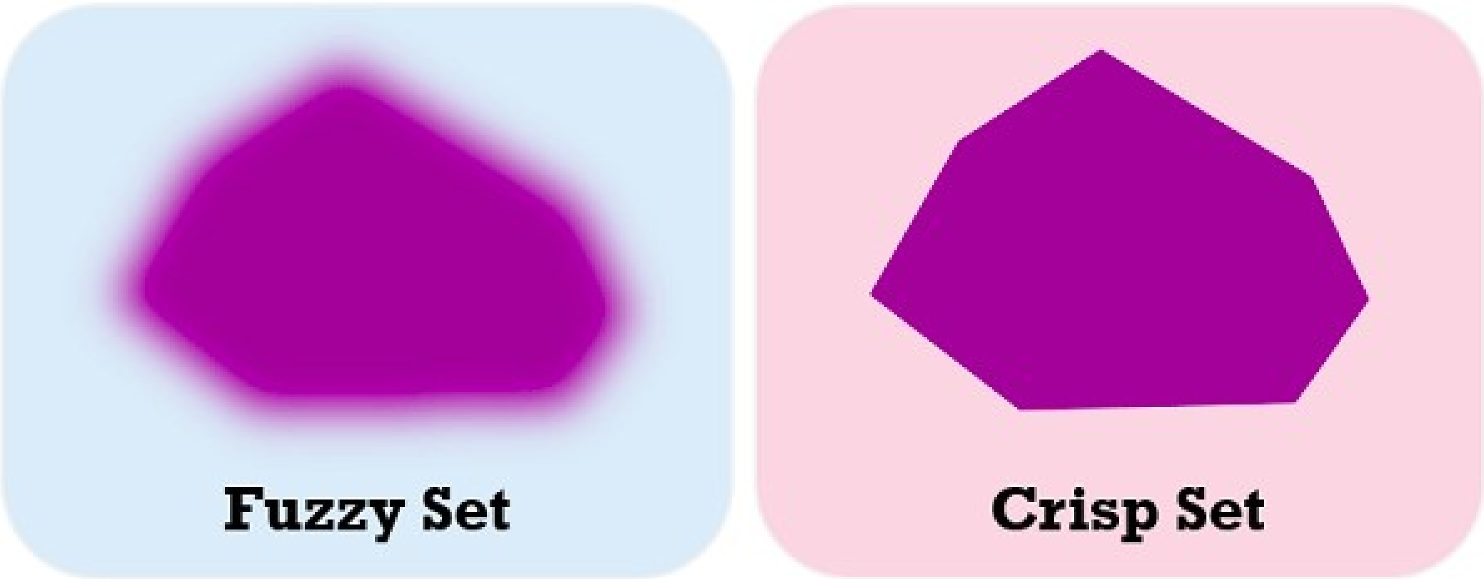


YES or NO

Difference:


Traditional (Crisp) Set Representation
Degree of Membership
1
0
Cold
Hot
70° F
temperature
Fuzzy Set Representation
Degree of Membership
1
0
Cold
Hot
70° F

Difference:

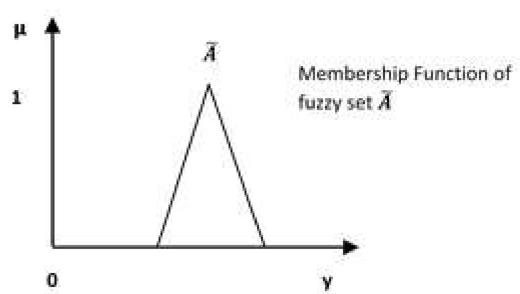


Difference:

BASIS FOR COMPARISON	FUZZY SET	CRISP SET
Basic	Prescribed by vague or ambiguous properties.	Defined by precise and certain characteristics.
Property	Elements are allowed to be partially included in the set.	Element is either the member of a set or not.
Applications	Used in fuzzy controllers	Digital design
Logic	Infinite-valued	bi-valued

Membership Function

- We already know that fuzzy logic is not logic that is fuzzy but logic that is used to describe fuzziness.
- This fuzziness is best characterized by its membership function.
- In other words, we can say that membership function represents the degree of truth in fuzzy logic.



Membership Function

Membership Function

- Following are a few important points relating to the membership function –
  - Membership functions were first introduced in 1965 by Lofti A. Zadeh in his first research paper “fuzzy sets”.
  - Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.
  - Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.
  - Membership functions are represented by graphical forms.
  - Rules for defining fuzziness are fuzzy too.

Mathematical Notation

- We have already studied that a fuzzy set Ã in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as –

$$\widetilde{A} = \left\{ \left(y, \mu_{\widetilde{A}}\left(y \right) \right) \middle| y \in U \right\}$$

- Here μA˜(·) = membership function of A˜; this assumes values in the range from 0 to 1, i.e., μA˜(·)∈[0,1]. The membership function μA˜(·) maps U to the membership space M.
- The dot (·) in the membership function described above, represents the element in a fuzzy set; whether it is discrete or continuous.

Features of Membership Functions

- Core
  - For any fuzzy set A˜, the core of a membership function is that region of universe that is characterize by full membership in the set.
  - Hence, core consists of all those elements y of the universe of information such that,

$$\mu_{\tilde{A}}(y) = 1$$

Features of Membership Functions

- Support
  - For any fuzzy set A˜, the support of a membership function is the region of universe that is characterize by a nonzero membership in the set.
  - Hence core consists of all those elements y of the universe of information such that,

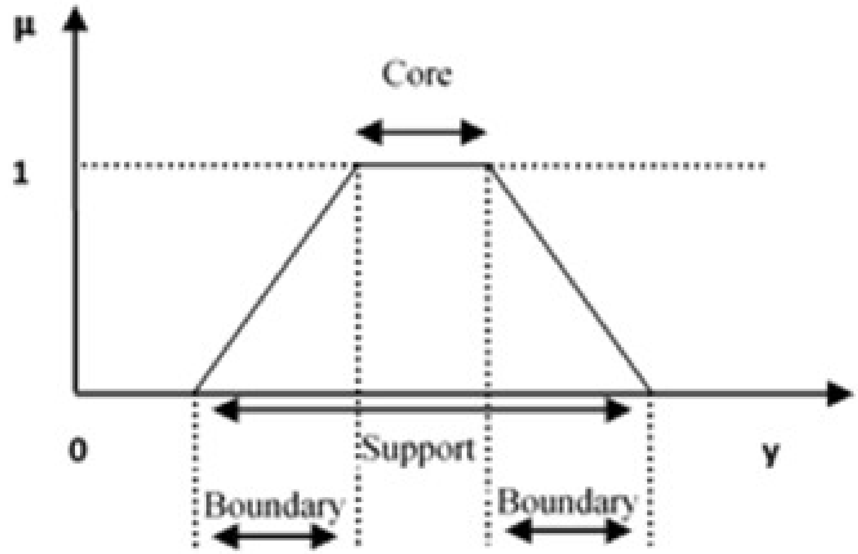
$$\mu_{\tilde{A}}(y) > 0$$

Features of Membership Functions

- Boundary
  - For any fuzzy set A˜ , the boundary of a membership function is the region of universe that is characterized by a nonzero but incomplete membership in the set.
  - Hence, core consists of all those elements y of the universe of information such that,

$$1 > \mu_{\tilde{A}}(y) > 0$$

Features of Membership Functions



Features of Membership Function

Fuzzification

- It may be defined as the process of transforming a crisp set to a fuzzy set or a fuzzy set to fuzzier set.
- Basically, this operation translates accurate crisp input values into linguistic variables.

Support Fuzzification(s-fuzzification) Method

- In this method, the fuzzified set can be expressed with the help of the following relation –

$$\widetilde{A} = \mu_1 Q(x_1) + \mu_2 Q(x_2) + \ldots + \mu_n Q(x_n)$$

- Here the fuzzy set Q(xi) is called as kernel of fuzzification.
- This method is implemented by keeping μi constant and xi being transformed to a fuzzy set Q(xi).

Grade Fuzzification (g-fuzzification)

- It is quite similar to the above method but the main difference is that it kept x_i constant and μ_i is expressed as a fuzzy set.

Defuzzification

- It may be defined as the process of reducing a fuzzy set into a crisp set or to convert a fuzzy member into a crisp member.
- We have already studied that the fuzzification process involves conversion from crisp quantities to fuzzy quantities.
- In a number of engineering applications, it is necessary to defuzzify the result or rather "fuzzy result" so that it must be converted to crisp result.
- Mathematically, the process of Defuzzification is also called "rounding it off".

Max-Membership Method

- This method is limited to peak output functions and also known as height method. Mathematically it can be represented as follows –

$$\mu_{\tilde{A}}(x^*) > \mu_{\tilde{A}}(x) \ \ for\ all\ x \in X$$

- Here, x∗ is the defuzzified output.

Centroid Method

- This method is also known as the center of area or the center of gravity method.
- Mathematically, the defuzzified output x∗ will be represented as –

$$x^* = \frac{\int \mu_{\tilde{A}}(x)\,.\,x\,dx}{\int \mu_{\tilde{A}}(x)\,.\,dx}$$

Weighted Average Method

- In this method, each membership function is weighted by its maximum membership value.
- Mathematically, the defuzzified output x∗ will be represented as –

$$x^* = \frac{\sum \mu_{\widetilde{A}}\left(\overline{x_i}\right).\overline{x_i}}{\sum \mu_{\widetilde{A}}\left(\overline{x_i}\right)}$$

Mean-Max Membership

- This method is also known as the middle of the maxima. Mathematically, the defuzzified output x∗ will be represented as –

$$x^* = \frac{\sum_{i=1}^{n} \overline{x_i}}{n}$$

Fuzzy Relation

- Fuzzy relation defines the mapping of variables from one fuzzy set to another.
- Like crisp relation, we can also define the relation over fuzzy sets.
- Let A be a fuzzy set on universe X and B be a fuzzy set on universe Y, then the Cartesian product between fuzzy sets A and B will result in a fuzzy relation R which is contained with the full Cartesian product space or it is subset of cartesian product of fuzzy subsets.

Fuzzy Relation

- Formally, we can define fuzzy relation as,

- $R = A \times B$

  and
- $R \subset (X \times Y)$

  where the relation R has membership function,
- $\mu R(x, y) = \mu A \times B(x, y) = \min(\mu A(x), \mu B(y))$

Fuzzy Relation

- A binary fuzzy relation $R(X, Y)$ is called bipartite graph if $X \neq Y$.
- A binary fuzzy relation $R(X, Y)$ is called directed graph or digraph if $X = Y$. , which is denoted as $R(X, X) = R(X^2)$

Fuzzy Relation

- Let A = {a1, a2, …, an} and B = {b1, b2, .., bm}, then fuzzy relation between A and B is described by the fuzzy relation matrix as,

$$\begin{bmatrix} \mu_{R(a_1,b_1)} & \mu_{R(a_1,b_2)} & \cdot & \cdot & \mu_{R(a_1,b_m)} \\ \mu_{R(a_2,b_1)} & \mu_{R(a_2,b_2)} & \cdot & \cdot & \mu_{R(a_2,b_m)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \mu_{R(a_n,b_1)} & \mu_{R(a_n,b_2)} & \cdot & \cdot & \mu_{R(a_n,b_m)} \end{bmatrix}$$

- We can also consider fuzzy relation as a mapping from the cartesian space (X, Y) to the interval [0, 1]. The strength of this mapping is represented by the membership function of the relation for every tuple μR(x, y)

Fuzzy Relation

- Given A = { (a1, 0.2), (a2, 0.7), (a3, 0.4) } and B = { (b1, 0.5), (b2, 0.6)}, find the relation over A x B
- Cartesian Product

$$\bar{R} = \bar{A} \times \bar{B} = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

Fuzzy Relation

- Fuzzy relations are very important because they can describe interactions between variables.
- Example: A simple example of a binary fuzzy relation on X = {1, 2, 3}, called ”approximately equal” can be defined as

  R(1, 1) = R(2, 2) = R(3, 3) = 1

  R(1, 2) = R(2, 1) = R(2, 3) = R(3, 2) = 0.8

  R(1, 3) = R(3, 1) = 0.3

Fuzzy Relation

- The membership function and relation matrix of R is given by,

$$\bar{R}(x, y) = \begin{cases} 1, & if\, x = y \\ 0.7, & if\, |x - y| = 1 \\ 0.3, & if\, |x - y| = 2 \end{cases}$$

R¯(x,y) = ⌊ ı ⎨ ı ⌈1,0.7,0.3,

ifx = yif|x - y| = 1if|x - y| = 2

$$\bar{R} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1.0 & 0.7 & 0.3 \\ 0.7 & 1.0 & 0.7 \\ 0.3 & 0.7 & 1.0 \end{bmatrix} \end{array}$$

Fuzzy Relation Operations

1. Union:

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \max[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)]$$

2. Intersection:

$$\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x, y) = \min[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)]$$

3. Complement:

$$\mu_{\overline{\underset{\sim}{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y)$$

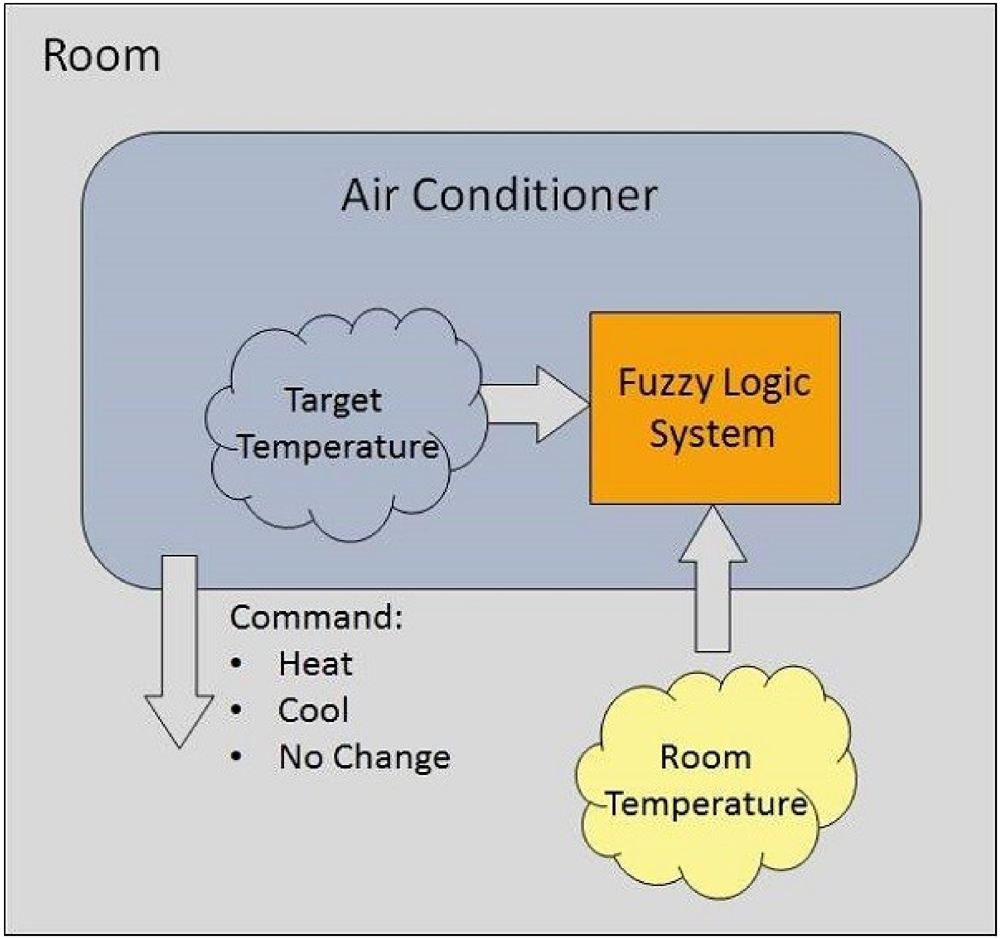
4. Containment:

$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \leq \mu_{\underset{\sim}{S}}(x, y)$$

Fuzzy Rule Based Systems

- Let us consider an air conditioning system with 5-level fuzzy logic system.
- This system adjusts the temperature of air conditioner by comparing the room temperature and the target temperature value.

Fuzzy Rule Based Systems



Fuzzy Rule Based Systems

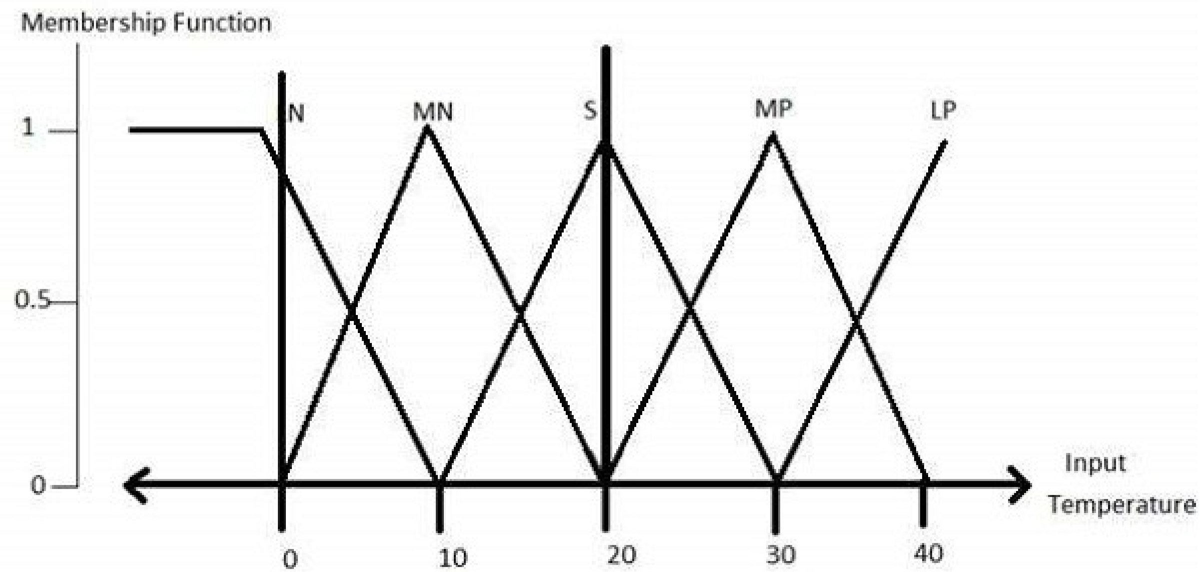
- Define linguistic Variables and terms (start)
- Construct membership functions for them. (start)
- Construct knowledge base of rules (start)
- Convert crisp data into fuzzy data sets using membership functions. (fuzzification)
- Evaluate rules in the rule base. (Inference Engine)
- Combine results from each rule. (Inference Engine)
- Convert output data into non-fuzzy values. (defuzzification)

Development

- Step 1 – Define linguistic variables and terms
  - Linguistic variables are input and output variables in the form of simple words or sentences. For room temperature, cold, warm, hot, etc., are linguistic terms.
  - Temperature (t) = {very-cold, cold, warm, very-warm, hot}
  - Every member of this set is a linguistic term and it can cover some portion of overall temperature values.

Development

- Step 2 – Construct membership functions for them
- The membership functions of temperature variable are as shown –



Development

- Step3 – Construct knowledge base rules
  - Create a matrix of room temperature values versus target temperature values that an air conditioning system is expected to provide.

RoomTemp. /Target	Very_Cold	Cold	Warm	Hot	Very_Hot
Very_Cold	No_Change	Heat	Heat	Heat	Heat
Cold	Cool	No_Change	Heat	Heat	Heat
Warm	Cool	Cool	No_Change	Heat	Heat
Hot	Cool	Cool	Cool	No_Change	Heat
Very_Hot	Cool	Cool	Cool	Cool	No_Change

Development

- Build a set of rules into the knowledge base in the form of IF-THEN-ELSE structures.

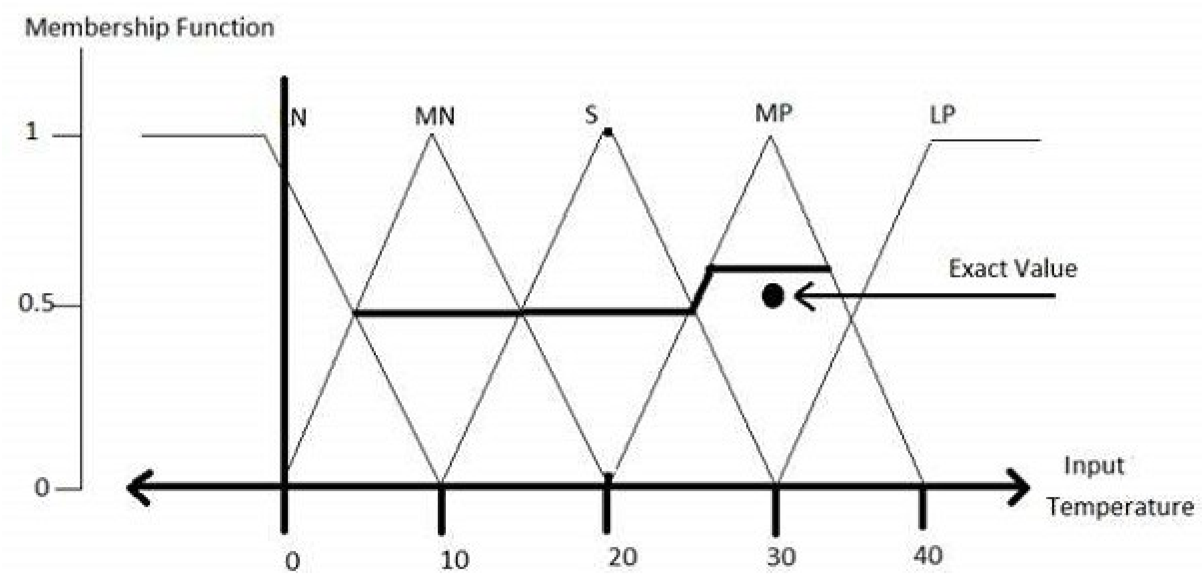
Sr. No.	Condition	Action
1	IF temperature=(Cold OR Very_Cold) AND target=Warm THEN	Heat
2	IF temperature=(Hot OR Very_Hot) AND target=Warm THEN	Cool
3	IF (temperature=Warm) AND (target=Warm) THEN	No_Change

Development

- Step 4 – Obtain fuzzy value
  - Fuzzy set operations perform evaluation of rules. The operations used for OR and AND are Max and Min respectively.
  - Combine all results of evaluation to form a final result. This result is a fuzzy value.

Development

- Step 5 – Perform defuzzification
  - Defuzzification is then performed according to membership function for output variable.



First Order Logic

- Consider the following sentence, which we cannot represent using PL logic.
  - "Some humans are intelligent", or
  - "Sachin likes cricket."
- To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

First Order Logic

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as Predicate logic or First-order predicate logic.
- First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

First Order Logic

- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
  - Objects: A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
  - Relations: It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
  - Function: Father of, best friend, third inning of, end of, ...
- As a natural language, first-order logic also has two main parts:
  - Syntax
  - Semantics

First Order Logic

- Following are the basic elements of FOL syntax:
  - Constant 1, 2, A, John, Mumbai, cat,....
  - Variables x, y, z, a, b,....
  - Predicates Brother, Father, >,....
  - Function sqrt, LeftLegOf, ...
  - Connectives ∧, ∨, ¬, ⇒, ⇔
  - Equality ==
  - Quantifier ∀, ∃

First Order Logic

- Atomic sentences:
  - Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
  - We can represent atomic sentences as Predicate (term1, term2, ..., term n).
  - Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).

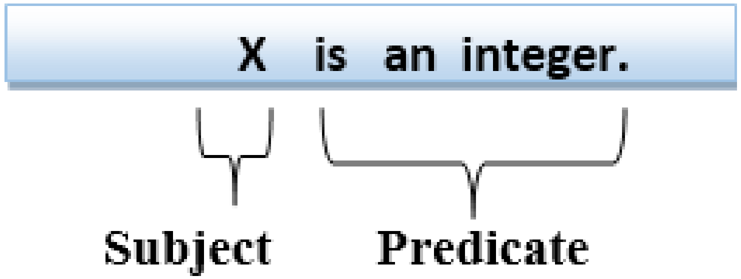
    Chinky is a cat: => cat (Chinky).

First Order Logic

- Complex Sentences:
  - Complex sentences are made by combining atomic sentences using connectives.
- First-order logic statements can be divided into two parts:
  - Subject: Subject is the main part of the statement.
  - Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.

First Order Logic

- Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



Predicate Logic

- Predicate Logic deals with predicates, which are propositions, consist of variables.
- A predicate is an expression of one or more variables determined on some specific domain.
- A predicate with variables can be made a proposition by either authorizing a value to the variable or by quantifying the variable.

Predicate Logic

- Examples:
  - Consider E(x, y) denote "x = y"
  - Consider X(a, b, c) denote "a + b + c = 0"
  - Consider M(x, y) denote "x is married to y."

Predicate Logic

- The variable of predicates is quantified by quantifiers.
- There are two types of quantifier in predicate logic - Existential Quantifier and Universal Quantifier.

Existential Quantifier

- If $p(x)$ is a proposition over the universe U. Then it is denoted as $\exists x\, p(x)$ and read as "There exists at least one value in the universe of variable x such that $p(x)$ is true.
- The quantifier $\exists$ is called the existential quantifier.
- There are several ways to write a proposition, with an existential quantifier, i.e.,

  $(\exists x \in A)p(x)$ or $\exists x \in A$ such that $p(x)$ or $(\exists x)p(x)$ or $p(x)$ is true for some $x \in A$.

Universal Quantifier

- If p(x) is a proposition over the universe U. Then it is denoted as $\forall x, p(x)$ and read as "For every $x \in U, p(x)$ is true."
- The quantifier $\forall$ is called the Universal Quantifier.
- There are several ways to write a proposition, with a universal quantifier.
- $\forall x \in A, p(x)$ or $p(x), \forall x \in A$ Or $\forall x, p(x)$ or p(x) is true for all $x \in A$.

Negation of Quantified Propositions

- When we negate a quantified proposition, i.e., when a universally quantified proposition is negated, we obtain an existentially quantified proposition,and when an existentially quantified proposition is negated, we obtain a universally quantified proposition.

Fuzzy Decision Making

- It is an activity which includes the steps to be taken for choosing a suitable alternative from those that are needed for realizing a certain goal.
- Steps involved in the decision making process –
  - Determining the Set of Alternatives – In this step, the alternatives from which the decision has to be taken must be determined.
  - Evaluating Alternative – Here, the alternatives must be evaluated so that the decision can be taken about one of the alternatives.
  - Comparison between Alternatives – In this step, a comparison between the evaluated alternatives is done.

Individual Decision Making

- In this type of decision making, only a single person is responsible for taking decisions. The decision making model in this kind can be characterized as –
  - Set of possible actions
  - Set of goals $G_i (i \in X_n)$;
  - Set of Constraints $C_j (j \in X_m)$

Individual Decision Making

- The goals and constraints stated above are expressed in terms of fuzzy sets.
- Now consider a set A. Then, the goal and constraints for this set are given by –

$$G_i(a) = \text{composition } [G_i(a)] = G_i^1(G_i(a)) \text{ with } G_i^1$$

$$C_j(a) = \text{composition } [C_j(a)] = C_j^1(C_j(a)) \text{ with } C_j^1 \quad \text{for } a \in A$$

- The fuzzy decision in the above case is given by –

$$F_D = min[i \in X_n^{in} f G_i(a), j \in X_m^{in} f C_j(a)]$$

Multi-Person Decision Making

- Decision making in this case includes several persons so that the expert knowledge from various persons is utilized to make decisions.
- Calculation for this can be given as follows –
- Number of persons preferring xi to xj = N(xi,xj)
- Total number of decision makers = n
- Then,

$$SC\left(x_i,\ x_j\right) = \frac{N(x_i,\ x_j)}{n}$$

Multi-objective Decision Making

- Multi-objective decision making occurs when there are several objectives to be realized. There are following two issues in this type of decision making –
  - To acquire proper information related to the satisfaction of the objectives by various alternatives.
  - To weigh the relative importance of each objective.

Multi-objective Decision Making

- Mathematically we can define a universe of n alternatives as –

  A=[a1,a2,...,ai,...,an]

- And the set of “m” objectives a O=[o1,o2,...,oi,...,on]

Multi-attribute Decision Making

- Multi-attribute decision making takes place when the evaluation of alternatives can be carried out based on several attributes of the object.
- The attributes can be numerical data, linguistic data and qualitative data.
- Mathematically, the multi-attribute evaluation is carried out on the basis of linear equation as follows –

$$Y = A1X1 + A2X2 + \ldots + AiXi + \ldots + ArXr$$

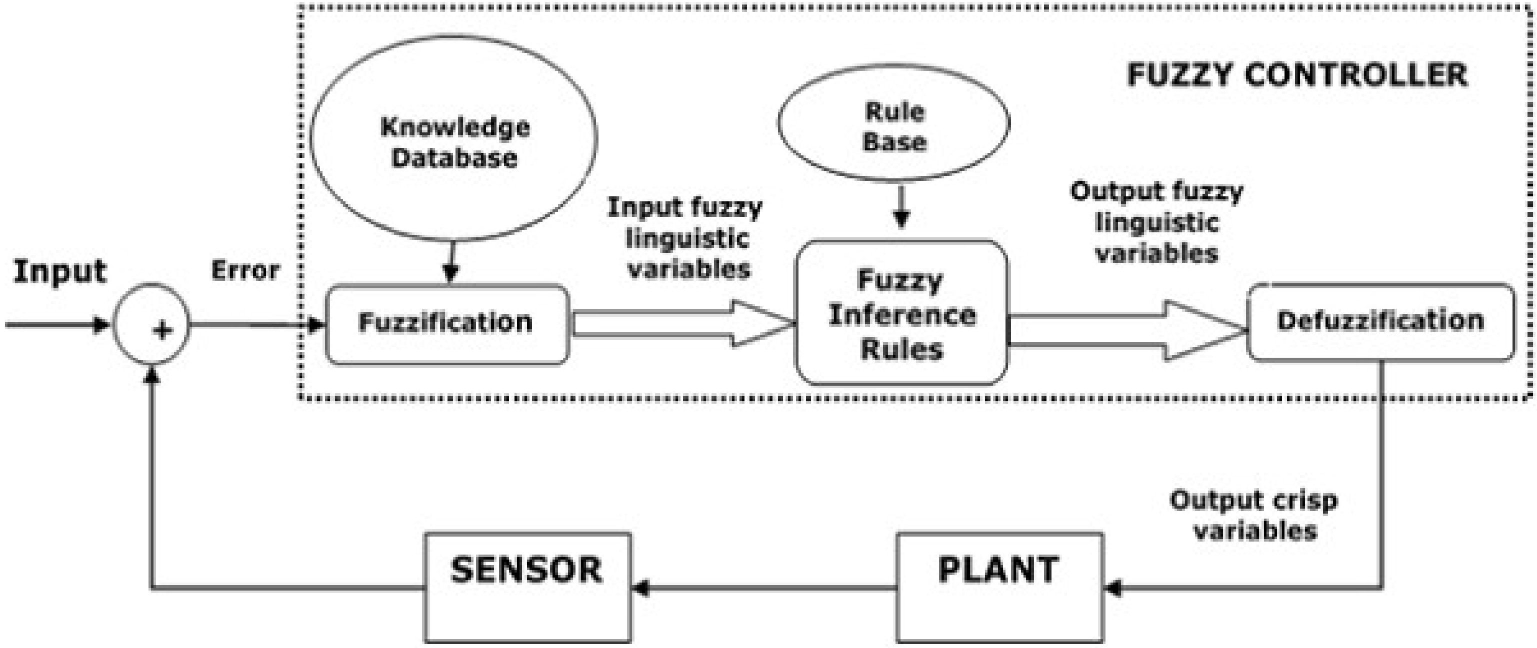
Fuzzy Control System

- Fuzzy logic is applied with great success in various control application. Almost all the consumer products have fuzzy control.
- Some of the examples include controlling your room temperature with the help of air-conditioner, anti-braking system used in vehicles, control on traffic lights, washing machines, large economic systems, etc.

Fuzzy Control System – Why?

- A control system is an arrangement of physical components designed to alter another physical system so that this system exhibits certain desired characteristics. Following are some reasons of using Fuzzy Logic in Control Systems –
  - While applying traditional control, one needs to know about the model and the objective function formulated in precise terms. This makes it very difficult to apply in many cases.
  - By applying fuzzy logic for control we can utilize the human expertise and experience for designing a controller.
  - The fuzzy control rules, basically the IF-THEN rules, can be best utilized in designing a controller.

Fuzzy Control System – Architecture



Major Components

- Fuzzifier – The role of fuzzifier is to convert the crisp input values into fuzzy values.
- Fuzzy Knowledge Base – It stores the knowledge about all the input-output fuzzy relationships. It also has the membership function which defines the input variables to the fuzzy rule base and the output variables to the plant under control.
- Fuzzy Rule Base – It stores the knowledge about the operation of the process of domain.
- Inference Engine – It acts as a kernel of any FLC. Basically it simulates human decisions by performing approximate reasoning.
- Defuzzifier – The role of defuzzifier is to convert the fuzzy values into crisp values getting from fuzzy inference engine.

Design Steps

- Identification of variables – Here, the input, output and state variables must be identified of the plant which is under consideration.
- Fuzzy subset configuration – The universe of information is divided into number of fuzzy subsets and each subset is assigned a linguistic label. Always make sure that these fuzzy subsets include all the elements of universe.
- Obtaining membership function – Now obtain the membership function for each fuzzy subset that we get in the above step.

Design Steps

- Fuzzy rule base configuration – Now formulate the fuzzy rule base by assigning relationship between fuzzy input and output.
- Fuzzification – The fuzzification process is initiated in this step.
- Combining fuzzy outputs – By applying fuzzy approximate reasoning, locate the fuzzy output and merge them.
- Defuzzification – Finally, initiate defuzzification process to form a crisp output.

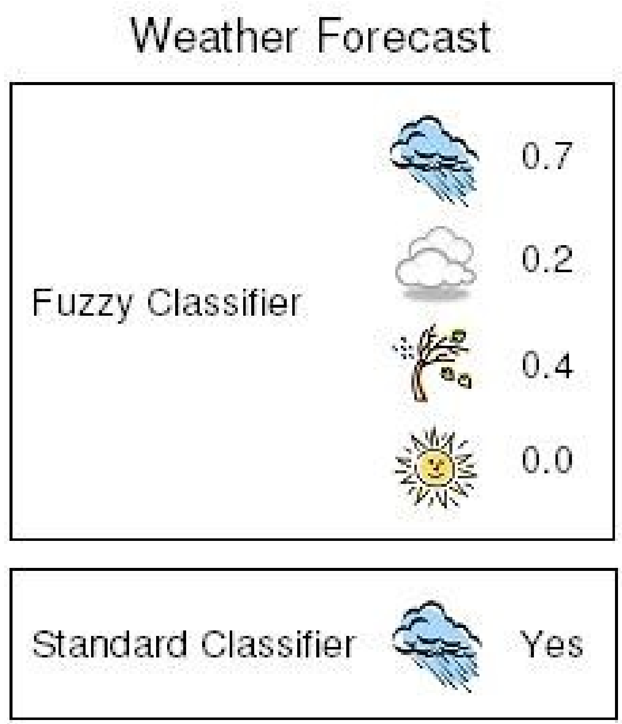
Advantages of FLC

- Cheaper – Developing a FLC is comparatively cheaper than developing model based or other controller in terms of performance.
- Robust – FLCs are more robust than PID controllers because of their capability to cover a huge range of operating conditions.
- Customizable – FLCs are customizable.
- Emulate human deductive thinking – Basically FLC is designed to emulate human deductive thinking, the process people use to infer conclusion from what they know.
- Reliability – FLC is more reliable than conventional control system.
- Efficiency – Fuzzy logic provides more efficiency when applied in control system.

Fuzzy Classification

- A classifier is an algorithm that assigns a class label to an object, based on the object description. It is also said that the classifier predicts the class label.
- The object description comes in the form of a vector containing values of the features (attributes) deemed to be relevant for the classification task.
- Typically, the classifier learns to predict class labels using a training algorithm and a training data set.
- When a training data set is not available, a classifier can be designed from prior knowledge and expertise.
- Once trained, the classifier is ready for operation on unseen objects.

Fuzzy Classification



Fuzzy rule-based classifiers

- Class label as the consequent
  - The simplest fuzzy rule-based classifier is a fuzzy if-then system, similar to that used in fuzzy control. Consider a 2D example with 3 classes. A fuzzy classifier can be constructed by specifying classification rules, e.g.,
  - IF x1 is medium AND x2 is small THEN class is 1
- IF x1 is medium AND x2 is large THEN class is 2
- IF x1 is large AND x2 is small THEN class is 2
- IF x1 is small AND x2 is large THEN class is 3

Fuzzy Decision Making

- It is an activity which includes the steps to be taken for choosing a suitable alternative from those that are needed for realizing a certain goal.
- Steps for Decision Making
-
- Steps involved in the decision making process –
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References:

- https://wisdomplexus.com

Thank you

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/mITuSkillologies

@mitu_group

/company/mitu-skillologies

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Web Resources
https://mitu.co.in
http://tusharkute.com

contact@mitu.co.in

tushar@tusharkute.com