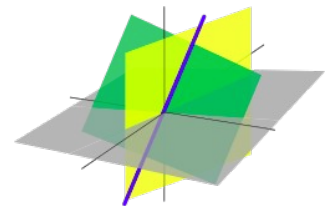


Linear Algebra

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Linear Algebra

- Linear algebra is a branch of mathematics that deals with linear equations and their representations in the vector space using matrices.
- In other words, linear algebra is the study of linear functions and vectors.
- It is one of the most central topics of mathematics.
- Most modern geometrical concepts are based on linear algebra.

Linear Algebra

- Linear algebra facilitates the modeling of many natural phenomena and hence, is an integral part of engineering and physics.
- Linear equations, matrices, and vector spaces are the most important components of this subject.

Linear Algebra

- The branch of mathematics that deals with vectors, matrices, finite or infinite dimensions as well as a linear mapping between such spaces is defined as linear algebra.
- It is used in both pure and applied mathematics along with different technical forms such as physics, engineering, natural sciences, etc.

Linear Algebra: Branches

- Linear algebra can be categorized into three branches depending upon the level of difficulty and the kind of topics that are encompassed within each.
- These are
 - elementary,
 - advanced, and
 - applied linear algebra.

Elementary Linear Algebra

- Elementary linear algebra introduces students to the basics of linear algebra.
- This includes simple matrix operations, various computations that can be done on a system of linear equations, and certain aspects of vectors.

Elementary Linear Algebra

- Scalars - A scalar is a quantity that only has magnitude and not direction. It is an element that is used to define a vector space. In linear algebra, scalars are usually real numbers.
- Vectors - A vector is an element in a vector space. It is a quantity that can describe both the direction and magnitude of an element.
- Vector Space - The vector space consists of vectors that may be added together and multiplied by scalars.

Elementary Linear Algebra

- Matrix - A matrix is a rectangular array wherein the information is organized in the form of rows and columns. Most linear algebra properties can be expressed in terms of a matrix.
- Matrix Operations - These are simple arithmetic operations such as addition, subtraction, and multiplication that can be conducted on matrices.

Advanced Linear Algebra

- Once the basics of linear algebra have been introduced to students the focus shifts on more advanced concepts related to linear equations, vectors, and matrices.
- Certain important terms that are used in advanced linear algebra are as follows:
 - Linear Transformations - The transformation of a function from one vector space to another by preserving the linear structure of each vector space.

Advanced Linear Algebra

- Inverse of a Matrix - When an inverse of a matrix is multiplied with the given original matrix then the resultant will be the identity matrix. Thus, $A^{-1}A = I$.
- Eigenvector - An eigenvector is a non-zero vector that changes by a scalar factor (eigenvalue) when a linear transformation is applied to it.
- Linear Map - It is a type of mapping that preserves vector addition and vector multiplication.

Advanced Linear Algebra

- Applied linear algebra is usually introduced to students at a graduate level in fields of applied mathematics, engineering, and physics.
- This branch of algebra is driven towards integrating the concepts of elementary and advanced linear algebra with their practical implications.
- Topics such as the norm of a vector, QR factorization, Schur's complement of a matrix, etc., fall under this branch of linear algebra.

Applications

- Signal Processing - Linear algebra is used in encoding and manipulating signals such as audio and video signals. Furthermore, it is required in the analysis of such signals.
- Linear Programming - It is an optimizing technique that is used to determine the best outcome of a linear function.
- Computer Science - Data scientists use several linear algebra algorithms to solve complicated problems.
- Prediction Algorithms - Prediction algorithms use linear models that are developed using concepts of linear algebra.

Scaler / Real Numbers

- Real number is any number that can be found in the real world. We find numbers everywhere around us.
- Natural numbers are used for counting objects, rational numbers are used for representing fractions, irrational numbers are used for calculating the square root of a number, integers for measuring temperature, and so on.
- These different types of numbers make a collection of real numbers.

Scaler / Real Numbers

- Any number that we can think of, except complex numbers, is a real number. For example, 3, 0, 1.5, $3/2$, $\sqrt{5}$, and so on are real numbers.
- Definition of Real Numbers
 - Real numbers include rational numbers like positive and negative integers, fractions, and irrational numbers.
 - Now, which numbers are not real numbers? The numbers that are neither rational nor irrational are non-real numbers, like, $\sqrt{-1}$, $2 + 3i$, and $-i$. These numbers include the set of complex numbers, C .

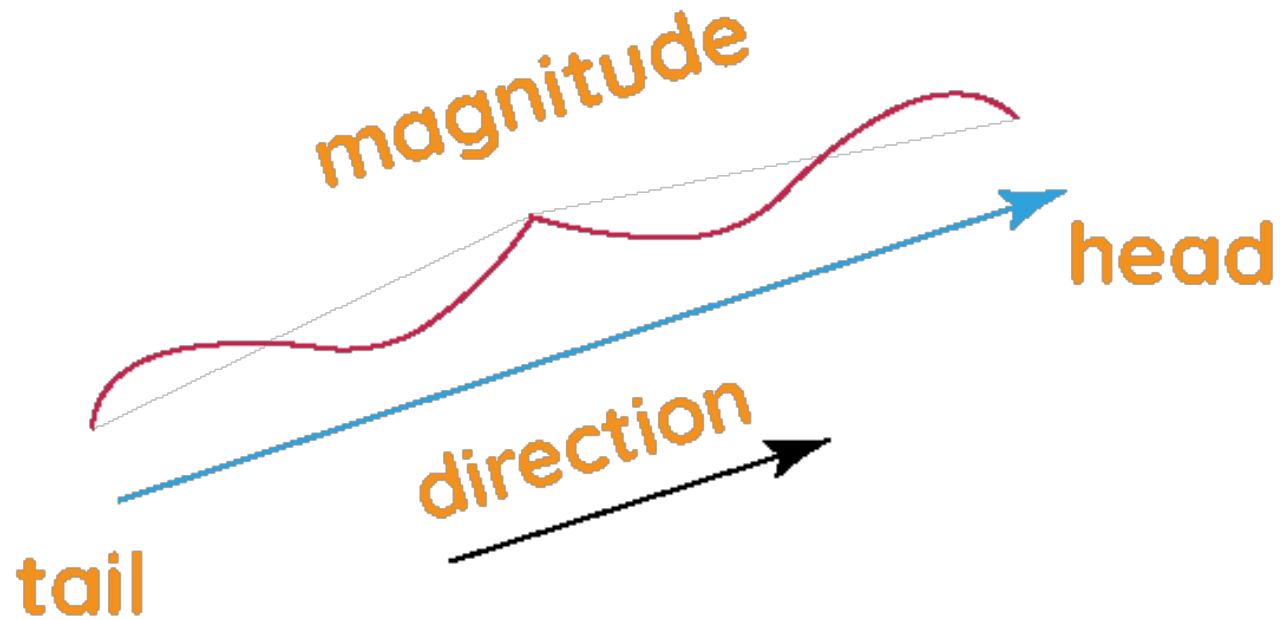
Vector

- Vectors are geometrical entities that have magnitude and direction.
- A vector can be represented by a line with an arrow pointing towards its direction and its length represents the magnitude of the vector.
- Therefore, vectors are represented by arrows, they have initial points and terminal points.
- The concept of vectors was evolved over a period of 200 years.
- Vectors are used to represent physical quantities such as displacement, velocity, acceleration, etc.

Vector

- A vector is a Latin word that means carrier. Vectors carry a point A to point B.
- The length of the line between the two points A and B is called the magnitude of the vector and the direction of the displacement of point A to point B is called the direction of the vector AB.
- Vectors are also called Euclidean vectors or Spatial vectors.
- Vectors have many applications in maths, physics, engineering, and various other fields.

Vector



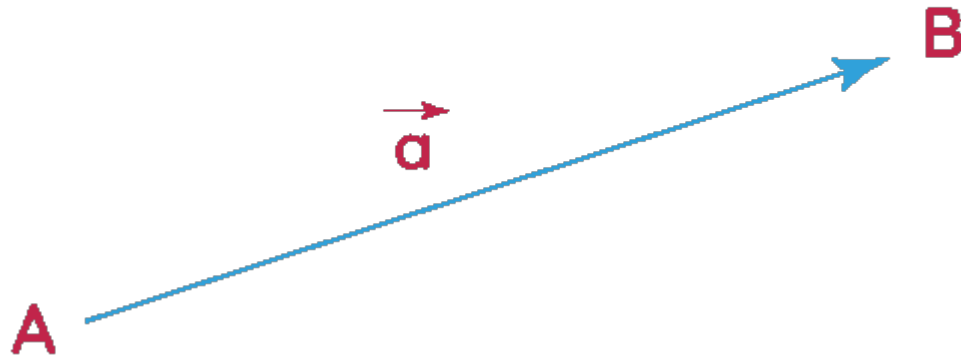
Vector in Euclidean Geometry

- Vectors in math is a geometric entity that has both magnitude and direction.
- Vectors have an initial point at the point where they start and a terminal point that tells the final position of the point.
- Various operations can be applied to vectors such as addition, subtraction, and multiplication.

Vector Representation

- Vectors are usually represented in bold lowercase such as \mathbf{a} or using an arrow over the letter as \vec{a} .
- Vectors can also be denoted by their initial and terminal points with an arrow above them, for example, vector AB can be denoted as \overrightarrow{AB} .
- The standard form of representation of a vector is $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$.
- Here, a, b, c are real numbers and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the x-axis, y-axis, and z-axis respectively.

Vector Representation



Matrices

- Matrices, the plural form of a matrix, are the arrangements of numbers, variables, symbols, or expressions in a rectangular table that contains various numbers of rows and columns.
- They are rectangular-shaped arrays, for which different operations like addition, multiplication, and transposition are defined.
- The numbers or entries in the matrix are known as its elements.
- Horizontal entries of matrices are called rows and vertical entries are known as columns.

Matrix

- A matrix is a rectangular array of numbers, variables, symbols, or expressions that are defined for the operations like subtraction, addition, and multiplications.
- The size of a matrix (which is known as the order of the matrix) is determined by the number of rows and columns in the matrix.
- The order of a matrix with 6 rows and 4 columns is represented as a 6×4 and is read as 6 by 4.

Matrix

- For example, the given matrix B is a 3×4 matrix and is written as $[B]_{3 \times 4}$

$$B = \begin{bmatrix} 2 & -1 & 3 & 5 \\ 0 & 5 & 2 & 7 \\ 1 & -1 & -2 & 9 \end{bmatrix}$$

Matrix

$$\begin{array}{c}
 \text{Columns} \\
 \left. \begin{array}{c} 1 \quad 2 \quad \dots \quad n \end{array} \right\} \\
 \left. \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} \right\} \text{Rows}
 \end{array}
 \left[\begin{array}{cccc}
 a_{11} & a_{12} & \dots & a_{1n} \\
 a_{21} & a_{22} & \dots & a_{2n} \\
 a_{31} & a_{32} & \dots & a_{3n} \\
 \vdots & \vdots & \dots & \vdots \\
 a_{m1} & a_{m2} & \dots & a_{mn}
 \end{array} \right] = A_{m \times n}$$

Matrix Notations

- If a matrix has m rows and n columns, then it will have $m \times n$ elements.
- A matrix is represented by the uppercase letter, in this case, 'A', and the elements in the matrix are represented by the lower case letter and two subscripts representing the position of the element in the number of row and column in the same order, in this case, ' a_{ij} ', where i is the number of rows, and j is the number of columns.

Matrix Notations

- For example, in the given matrix A, element in the 3rd row and 2nd column would be a_{32} , can be verified in the matrix given below:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \dots & a_{mn} \end{bmatrix}$$

Matrix Operations

- The basic operations that can be performed on matrices are:
 - Addition of Matrices
 - Subtraction of Matrices
 - Scalar Multiplication
 - Multiplication of Matrices
 - Transpose of Matrices

Matrix Addition

- The addition of matrices can only be possible if the number of rows and columns of both the matrices are the same.
- While adding 2 matrices, we add the corresponding elements. i.e.,
 - $(A + B) = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$,
where i and j are the number of rows and columns respectively.

Matrix Addition

- For example:

$$\begin{aligned} & \begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2+0 & -1+2 \\ 0+1 & 5+(-2) \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \end{aligned}$$

Matrix Subtraction

- Matrices subtraction is also possible only if the number of rows and columns of both the matrices are the same.
- While subtracting 2 matrices, we subtract the corresponding elements. i.e.,
 - $(A - B) = [a_{ij}] - [b_{ij}] = [a_{ij} - b_{ij}]$,
where i and j are the row number and column number respectively.

Matrix Substraction

- For example:

$$\begin{aligned} & \begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2-0 & -1-2 \\ 0-1 & 5-(-2) \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 \\ -1 & 7 \end{bmatrix} \end{aligned}$$

Scalar Multiplication

- The product of a matrix A with any number ' c ' is obtained by multiplying every entry of the matrix A by c , is called scalar multiplication. i.e., $(cA)_{ij} = c(A_{ij})$
- Properties of scalar multiplication in matrices:
- The different properties of matrices for scalar multiplication of any scalars K and l , with matrices A and B are given as,
 - $K(A + B) = KA + KB$
 - $(K + l)A = KA + lA$
 - $(Kl)A = K(lA) = l(KA)$
 - $(-K)A = -(KA) = K(-A)$
 - $1 \cdot A = A$
 - $(-1)A = -A$

Matrix Multiplication

- Matrices multiplication is defined only if the number of columns in the first matrix and rows in the second matrix are equal.

Matrix Multiplication

$$\begin{array}{c}
 \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{array} \right] \times \left[\begin{array}{cc} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{array} \right] = \left[\begin{array}{cc} a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{array} \right] \\
 3 \times 3 \qquad \qquad 3 \times 2 \qquad \qquad \qquad \qquad 3 \times 2
 \end{array}$$

$$\begin{array}{c}
 \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{array} \right] \times \left[\begin{array}{cc} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{array} \right] = \left[\begin{array}{cc} \boxed{} & a_1\beta_1 + a_2\beta_2 + a_3\beta_3 \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{array} \right] \\
 3 \times 3 \qquad \qquad 3 \times 2 \qquad \qquad \qquad \qquad 3 \times 2
 \end{array}$$

Matrix Multiplication

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \times \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{bmatrix} = \begin{bmatrix} \boxed{} & \boxed{} \\ b_1\alpha_1 + b_2\alpha_2 + b_3\alpha_3 & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

3×3 3×2 3×2

$$R_i \rightarrow \begin{bmatrix} \phantom{a_{i1}} & \phantom{a_{in}} \\ a_{i1} & \phantom{a_{in}} \\ \phantom{a_{i1}} & \phantom{a_{in}} \end{bmatrix} \times \begin{bmatrix} \phantom{b_{1i}} \\ b_{1i} \\ \phantom{b_{1i}} \\ \phantom{b_{ni}} \\ b_{ni} \end{bmatrix} = \begin{bmatrix} \phantom{a_{i1}} & \phantom{a_{in}} \\ \phantom{a_{i1}} & \phantom{a_{in}} \\ \phantom{a_{i1}} & \phantom{a_{in}} \end{bmatrix}$$

$m \times n$ $n \times p$ $m \times p$

Matrix Multiplication: Properties

- There are different properties associated with the multiplication of matrices.
- For any three matrices A, B, and C:
 - $AB \neq BA$
 - $A(BC) = (AB)C$
 - $A(B + C) = AB + AC$
 - $(A + B)C = AC + BC$
 - $AI_m = A = AI_n$, for identity matrices I_m and I_n .
 - $A_{m \times n}O_{n \times p} = O_{m \times p}$, where O is a null matrix.

Matrix Multiplication: Example

```
import numpy as np

# Create two matrices
A = np.array([[1, 2], [3, 4]])
B = np.array([[5, 6], [7, 8]])

# Calculate the product of the two matrices
C = np.matmul(A, B)

# Print the product
print(C)
```

Matrix Multiplication: Applications

- Matrix multiplication is a powerful operation that can be used to perform a variety of tasks, such as:
 - Solving systems of linear equations
 - Transforming vectors and matrices
 - Calculating statistics
 - Performing machine learning algorithms

Matrix Transpose

- The transpose of a matrix is done when we replace the rows of a matrix to the columns and columns to the rows.
- Interchanging of rows and columns is known as the transpose of matrices. In the matrix given below, we have row elements as row-1: 2, -3, -4, and row-2: -1, 7, -7.
- On transposing, we will get the elements in column-1: 2, -3, -4, and column-2: -1, 7, -7,

Matrix Transpose

Transpose of 2×3 Matrix

$$A_{2 \times 3} = \begin{bmatrix} 2 & -3 & -4 \\ -1 & 7 & -7 \end{bmatrix} \begin{array}{l} \leftarrow \text{Row 1} \\ \leftarrow \text{Row 2} \end{array}$$

$$A^T_{3 \times 2} = \begin{array}{cc} \text{Column 1} & \text{Column 2} \\ \downarrow & \downarrow \\ \begin{bmatrix} 2 & -1 \\ -3 & 7 \\ -4 & -7 \end{bmatrix} \end{array}$$

Matrix Transpose: Properties

- There are various properties associated with transposition. For matrices A and B, given as,
 - $(A^T)^T = A$
 - $(A + B)^T = A^T + B^T$, A and B being of the same order.
 - $(KA)^T = KA^T$, K is any scalar(real or complex).
 - $(AB)^T = B^T A^T$, A and B being conformable for the product AB. (This is also called reversal law.)

Matrix Transpose: Example

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the transpose of the matrix
B = np.transpose(A)

# Print the transpose
print(B)
```

Matrix Trace

- The trace of any matrix A , $\text{Tr}(A)$ is defined as the sum of its diagonal elements. Some properties of trace of matrices are,

$$\text{tr}(AB) = \text{tr}(BA)$$

$$\text{tr}(A) = \text{tr}(A^T)$$

$$\text{tr}(cA) = c \text{tr}(A), \text{ for a scalar 'c'}$$

$$\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$$

Matrix Trace

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the trace of the matrix
trace = np.trace(A)

# Print the trace
print(trace)
```

Determinant of Matrix

- The determinant of a matrix is a number defined only for square matrices.
- It is used in the analysis of linear equations and their solution.
- The determinant formula helps calculate the determinant of a matrix using the elements of the matrix.
- Determinant of a matrix is equal to the summation of the product of the elements of a particular row or column with their respective cofactors. Determinant of a matrix A is denoted as $|A|$.

Determinant of Matrix

- Let say we want to find the determinant of the matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Determinant of Matrix

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the determinant of the matrix
determinant = np.linalg.det(A)

# Print the determinant
print(determinant)
```

Inverse of Matrix

- The inverse of any matrix is denoted as the matrix raised to the power (-1), i.e. for any matrix "A", the inverse matrix is denoted as A^{-1} .
- The inverse of a square matrix, A is A^{-1} only when: $A \times A^{-1} = A^{-1} \times A = I$.
- There is a possibility that sometimes the inverse of a matrix does not exist if the determinant of the matrix is equal to zero ($|A| = 0$).
- The inverse of a matrix is shown by A^{-1} .

Inverse of Matrix

- Matrices inverse is calculated by using the following formula:

$$A^{-1} = (1/|A|)(\text{Adj } A)$$

- Where,
|A| is the determinant of the matrix A and $|A| \neq 0$.
Adj A is the adjoint of the given matrix A.

Inverse of Matrix

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the inverse of the matrix
A_inv = np.linalg.inv(A)

# Print the inverse of the matrix
print(A_inv)
```

Rank of matrix

- The rank of a matrix A is defined as the maximum number of linearly independent row(or column) vectors of the matrix.
- That means the rank of a matrix will always be less than or equal to the number of its rows or columns.
- The rank of a null matrix is zero since it has no independent row or column vectors.

Rank of matrix

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the rank of the matrix
rank = np.linalg.matrix_rank(A)

# Print the rank
print(rank)
```

Types of Matrix

- There are various types of matrices based on the number of elements and the arrangement of elements in them.
- Row matrix: A row matrix is a matrix having a single row is called a row matrix. Example: $[1, -2, 4]$.
- Column matrix: A column matrix is a matrix having a single column is called a column matrix.
- Example: $[-1, 2, 5]^T$.

Types of Matrix

- Square matrix: A matrix having equal number of rows and columns is called a square matrix. For example:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 6 & 1 \end{bmatrix}$$

- Rectangular Matrix: A matrix having unequal number of rows and columns is called a rectangular matrix. For example:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$$

Types of Matrix

- Diagonal matrices: A matrix with all non-diagonal elements to be zeros is known as a diagonal matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- Identity matrices: A diagonal matrix having all the diagonal elements equal to 1 is called an identity matrix.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Types of Matrix

- Symmetric matrices: A square matrix D of size $n \times n$ is considered to be symmetric if and only if $D^T = D$.

$$D = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 6 & 5 & 9 \end{bmatrix}$$

- Skew-symmetric matrices-A square matrix F of size $n \times n$ is considered to be skew-symmetric if and only if $F^T = -F$.

$$F = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$



$$\begin{aligned} \bullet F^T &= \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \\ \bullet -F &= \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \end{aligned}$$

Types of Matrix

- **Invertible Matrix:** Any square matrix A is called invertible matrix, if there exists another matrix B , such that, $AB = BA = I_n$, where I_n is an identity matrix with $n \times n$.
- **Orthogonal Matrix:** Any square matrix A is orthogonal if its transpose is equal to its inverse.
i.e., $A^T = A^{-1}$

Types of Matrix

- An orthogonal matrix is a square matrix whose transpose is equal to its inverse. In other words, if Q is an orthogonal matrix, then $Q^T=Q^{-1}$.
- Orthogonal matrices are also known as unitary matrices, because they preserve the magnitude of vectors.
- This means that if v is a vector and Q is an orthogonal matrix, then $||Qv||=||v||$.

Orthogonal Matrix

- Orthogonal matrices have many important properties, including the following:
 - The determinant of an orthogonal matrix is either 1 or -1.
 - The eigenvalues of an orthogonal matrix are all of magnitude 1.
 - The columns of an orthogonal matrix form an orthonormal basis, which means that they are all of magnitude 1 and are orthogonal to each other.

Orthonormal Matrix

- An orthonormal matrix is a square matrix whose columns (and rows) are orthonormal vectors.
- In other words, the columns of an orthonormal matrix are orthogonal (perpendicular) to each other and have a magnitude of 1.
- Orthonormal matrices have many important properties, including:
 - Their inverse is equal to their transpose.
 - Their determinant is equal to 1 or -1.
 - They can be used to represent rotations and reflections in Euclidean space.

Orthonormal Matrix

- Here is an example of an orthonormal matrix in two dimensions:

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

- This matrix represents a rotation of 45 degrees counterclockwise.
- Orthonormal matrices are a powerful tool for representing and manipulating vectors and matrices. They have many important applications in a wide range of fields.

Eigen values

- The eigenvalues of matrix are scalars by which some vectors (eigenvectors) change when the matrix (transformation) is applied to it.
- In other words, if A is a square matrix of order $n \times n$ and v is a non-zero column vector of order $n \times 1$ such that $Av = \lambda v$ (it means that the product of A and v is just a scalar multiple of v), then the scalar (real number) λ is called an eigenvalue of the matrix A that corresponds to the eigenvector v .

Eigen values

- The word "eigen" is from the German language and it means "characteristic", "proper", or "own".
- Thus, eigenvalues are also known as "characteristic values" (or) "characteristic roots" (or) "proper values", etc.

Eigen values

If A is a square matrix and V is a column vector such that:



$$AV = \lambda V$$

then



V = Eigen vector of A



λ = Eigen value of A

Eigen values

- From the definition of eigenvalues, if λ is an eigenvalue of a square matrix A , then

$$Av = \lambda v$$

- If I is the identity matrix of the same order as A , then we can write the above equation as

$$Av = \lambda (Iv) \text{ (because } v = Iv)$$

$$Av - \lambda (Iv) = 0$$

Taking v as common factor,

$$v (A - \lambda I) = 0$$

Eigen values

- This represents a homogeneous system of linear equations and it has a non-trivial solution only when the determinant of the coefficient matrix is 0.

i.e., $|A - \lambda I| = 0$

- This equation is called the characteristic equation (where $|A - \lambda I|$ is called the characteristic polynomial) and by solving this for λ , we get the eigenvalues.

Eigen values

- Here is the step-by-step process used to find the eigenvalues of a square matrix A .
 - Take the identity matrix I whose order is the same as A .
 - Multiply every element of I by λ to get λI .
 - Subtract λI from A to get $A - \lambda I$.
 - Find its determinant.
 - Set the determinant to zero and solve for λ .

Eigen values

- Let us see the process of finding the eigenvalues of a 2x2 matrix with an example where we will find the eigenvalues of A

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

- Let λ represents its eigenvalue(s). The identity matrix of order 2x2 is, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Eigen values

$$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 5 - \lambda & 4 \\ 1 & 2 - \lambda \end{bmatrix}$$

- Its determinant is,
- $|A - \lambda I| = (5 - \lambda)(2 - \lambda) - (1)(4)$
 $= 10 - 5\lambda - 2\lambda + \lambda^2 - 4$
 $= \lambda^2 - 7\lambda + 6$
- The characteristic equation is,
 $|A - \lambda I| = 0$
 $\lambda^2 - 7\lambda + 6 = 0$
 $(\lambda - 6)(\lambda - 1) = 0$
 $\lambda - 6 = 0; \lambda - 1 = 0$
 $\lambda = 6; \lambda = 1$
- Thus, the eigenvalues of matrix A are 1 and 6.

Eigen Vector and Values : Example

- Example: For this matrix

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$$

- an eigenvector is:

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

- with a matching eigenvalue of 6

Eigen Vector and Values : Example

- Av gives us:

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -6 \times 1 + 3 \times 4 \\ 4 \times 1 + 5 \times 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \end{bmatrix}$$

- λv gives us :

$$6 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \end{bmatrix}$$

- Yes they are equal! So $Av = \lambda v$ as promised.

Eigne values: Applications

- Eigenvalues are used in electric circuits, quantum mechanics, control theory, etc.
- They are used in the design of car stereo systems.
- They are also used to design bridges.
- It is not surprising to know that eigenvalues are also used in determining Google's page rank.
- They are used in geometric transformations.

Eigne values: Example

```
import numpy as np

# Create a matrix
A = np.array([[1, 2], [3, 4]])

# Calculate the eigenvalues and eigenvectors of the matrix
w, v = np.linalg.eig(A)

# Print the eigenvalues
print(w)

# Print the eigenvectors
print(v)
```

Thank you

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