Logic

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Propositional Logic

- Propositional logic, also known as propositional calculus or boolean logic, is a branch of logic that deals with propositions and their relationships through logical connectives.
- It forms the foundation of classical logic and is essential in fields like mathematics, computer science, and artificial intelligence.

Propositional Logic

- Propositions:
  - A proposition is a declarative sentence that is either true or false, but not both.
  - Examples:
    - "The sky is blue."
    - "2 + 2 = 4."
    - "It is raining."

Propositional Logic

- Logical Connectives:
  - Connectives are used to combine propositions to form more complex logical statements. The primary connectives are:
    - AND (∧): Conjunction; true if both propositions are true.
    - OR (∨): Disjunction; true if at least one of the propositions is true.
    - NOT (¬): Negation; true if the proposition is false.
    - IMPLIES (→): Implication; true if the first proposition implies the second.
    - IF AND ONLY IF ( ): Biconditional; true if both propositions are either true or false.

Propositional Logic

- Truth Tables:
  - A truth table shows the truth value of a proposition for every possible combination of truth values of its components.
- Example: Truth table for A∧BA∧B

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional Logic

- Tautologies, Contradictions, and Contingencies:
  - Tautology: A proposition that is always true, regardless of the truth values of its components.

    Example: A∨¬AA∨¬A
  - Contradiction: A proposition that is always false.

    Example: A∧¬AA∧¬A
  - Contingency: A proposition that is neither always true nor always false.

Propositional Logic

- Logical Equivalence:
  - Two propositions are logically equivalent if they have the same truth value in every possible situation.
  - Example: A∨BA∨B is equivalent to B∨AB∨A (commutative property).

Propositional Logic

- Example of Propositional Logic
- Let's consider a practical example: determining the truth value of a complex proposition.
  - Propositions:

    P: "It is raining."

    Q: "I have an umbrella."

    R: "I will not get wet."
  - Logical Statement:
    - $(P \to Q) \land (Q \to R)(P \to Q) \land (Q \to R)$

Propositional Logic

- Step-by-Step Analysis:
  - Identify the Components:

    P: "It is raining."

    Q: "I have an umbrella."

    R: "I will not get wet."

Propositional Logic

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Propositional Logic

- Analyze the Results:
  - The proposition (P→Q)∧(Q→R)(P→Q)∧(Q→R) is true in the following cases:
  - When PP is false and QQ is true, regardless of RR's value.
  - When PP and QQ are both true, and RR is true.

Predicate Logic

- Predicate logic, also known as first-order logic (FOL), extends propositional logic by dealing with predicates, which express properties of objects or relationships between objects.
- Predicate logic is more expressive than propositional logic, allowing for the representation of more complex statements and reasoning.

First Order Logic

- Consider the following sentence, which we cannot represent using PL logic.
  - "Some humans are intelligent", or
  - "Sachin likes cricket."
- To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

First Order Logic

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as Predicate logic or First-order predicate logic.
- First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

First Order Logic

- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
  - Objects: A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
  - Relations: It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
  - Function: Father of, best friend, third inning of, end of, ...
- As a natural language, first-order logic also has two main parts:
  - Syntax
  - Semantics

First Order Logic

- Following are the basic elements of FOL syntax:
  - Constant 1, 2, A, John, Mumbai, cat,....
  - Variables x, y, z, a, b,....
  - Predicates Brother, Father, >,....
  - Function sqrt, LeftLegOf, ...
  - Connectives ∧, ∨, ¬, ⇒, ⇔
  - Equality ==
  - Quantifier ∀, ∃

First Order Logic

- Atomic sentences:
  - Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
  - We can represent atomic sentences as Predicate (term1, term2, ..., term n).
  - Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).

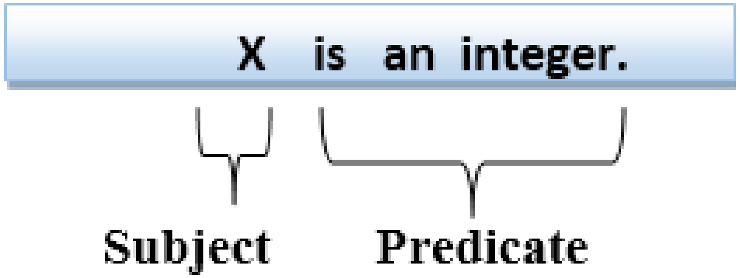
    Chinky is a cat: => cat (Chinky).

First Order Logic

- Complex Sentences:
  - Complex sentences are made by combining atomic sentences using connectives.
- First-order logic statements can be divided into two parts:
  - Subject: Subject is the main part of the statement.
  - Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.

First Order Logic

- Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



Quantifiers in First Order Logic

- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:
  - Universal Quantifier, (for all, everyone, everything)
  - Existential quantifier, (for some, at least one).

Universal Quantifier

- Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.
- The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.
- Note: In universal quantifier we use implication "→".
- If x is a variable, then ∀x is read as:

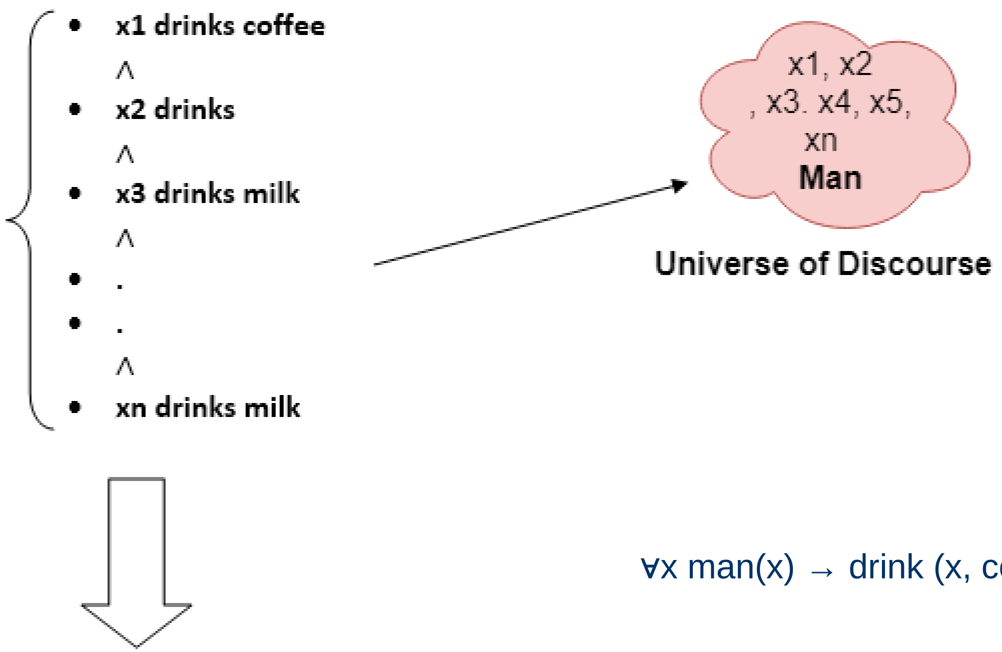
  For all x

  For each x

  For every x.

Universal Quantifier

- Example: All man drink coffee.
- Let a variable x which refers to a cat so all x can be represented in UOD as below:



So in shorthand notation, we can write it as :

$\forall x \; man(x) \rightarrow drink(x, coffee)$.

It will be read as: There are all x where x is a man who drink coffee.

Existential Quantifier

- Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.
- Note: In Existential quantifier we always use AND or Conjunction symbol (∧).
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:

    There exists a 'x.'

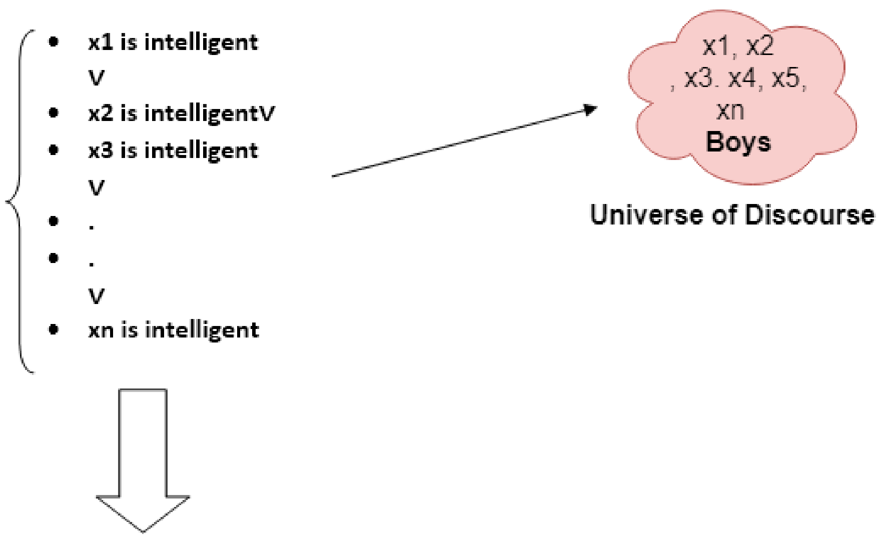
    For some 'x.'

    For at least one 'x.'

Existential Quantifier

- Example:

  Some boys are intelligent.



So in short-hand notation, we can write it as:

$\exists x: boys(x) \wedge intelligent(x)$

- It will be read as: There are some x where x is a boy who is intelligent.

Points

- Points to remember:
  - The main connective for universal quantifier ∀ is implication →.
  - The main connective for existential quantifier ∃ is and ∧.
- Properties of Quantifiers:
  - In universal quantifier, ∀x∀y is similar to ∀y∀x.
  - In Existential quantifier, ∃x∃y is similar to ∃y∃x.
  - ∃x∀y is not similar to ∀y∃x.

Examples

- 1. All birds fly.

  In this question the predicate is "fly(bird)."

  And since there are all birds who fly so it will be represented as follows.

  $$\forall x \; bird(x) \rightarrow fly(x).$$

- 2. Every man respects his parent.

  In this question, the predicate is "respect(x, y)," where x=man, and y= parent.

  Since there is every man so will use ∀, and it will be represented as follows:

  $$\forall x \; man(x) \rightarrow respects\,(x, parent).$$

Examples

- 3. Some boys play cricket.

  In this question, the predicate is "play(x, y)," where x= boys, and y= game. Since there are some boys so we will use ∃, and it will be represented as:

  $$\exists x \; boys(x) \rightarrow play(x, cricket).$$

- 4. Not all students like both Mathematics and Science.

  In this question, the predicate is "like(x, y)," where x= student, and y= subject.

  Since there are not all students, so we will use ∀ with negation, so following representation for this:

  $$\neg \forall (x) \; [\; student(x) \rightarrow like(x, Mathematics) \wedge like(x, Science)].$$

Examples

- 5. Only one student failed in Mathematics.

  In this question, the predicate is "failed(x, y)," where x= student, and y= subject.

  Since there is only one student who failed in Mathematics, so we will use following representation for this:

  $$\exists(x)\ [\ student(x) \rightarrow failed\ (x, Mathematics)\ \land \forall\ (y)\ [\neg(x==y)\ \land\ student(y) \rightarrow \neg failed\ (x, Mathematics)].$$

Thank you

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